Deep Reinforcement Learning

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Deep Structured Learning Course
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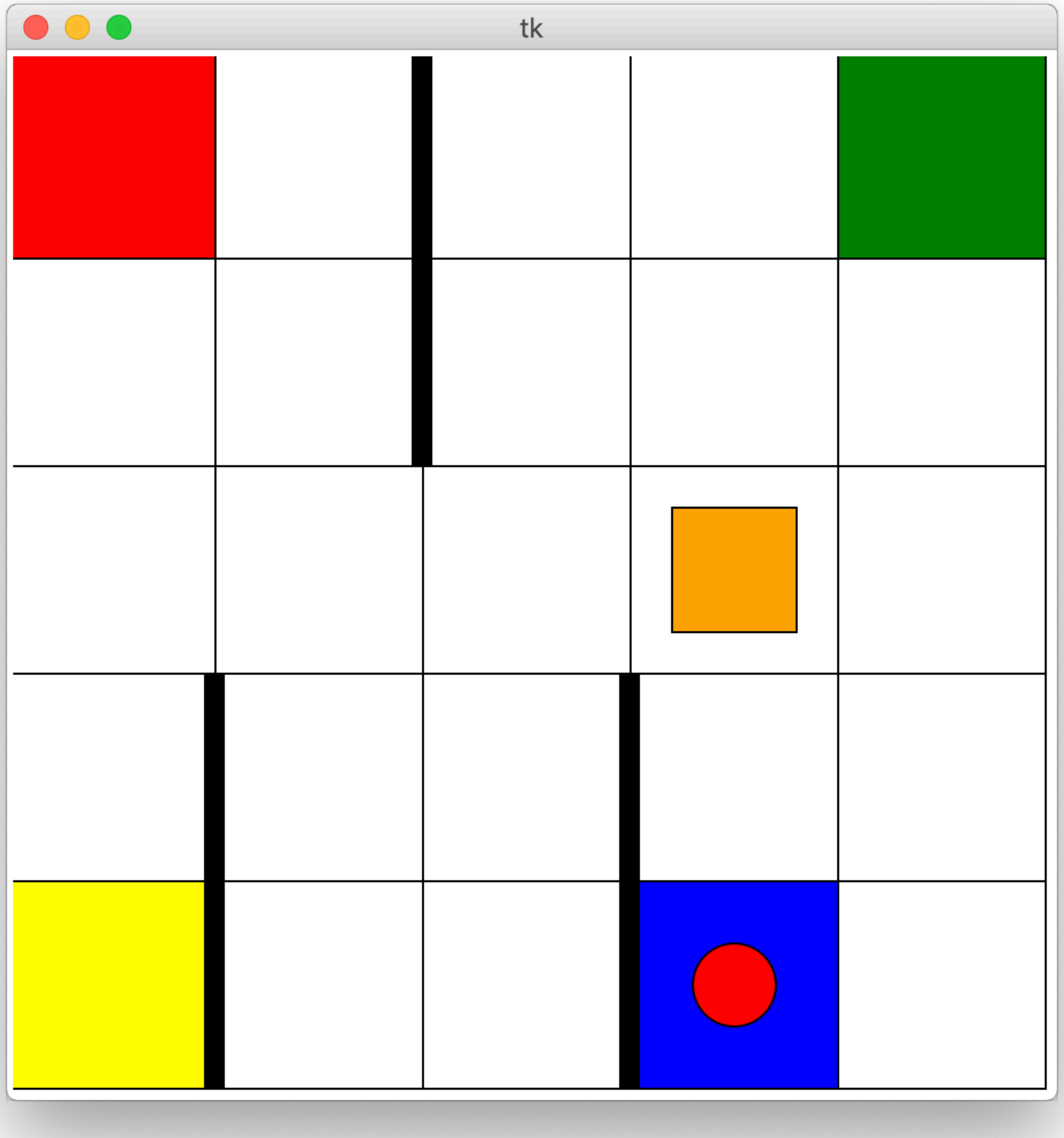
Outline of the lecture

- **Part I: RL Primer**
    - The RL Problem
    - Markov Decision Process - A Model for RL Problems
    - Optimality & Dynamic Programming
    - Monte Carlo Approaches
    - Temporal Difference Learning
    - The Policy Gradient Theorem

Outline of the lecture

- **Part II: Deep RL**
  - From RL to Deep RL
  - DQN
  - Deep advantage actor-critic methods
  - Trust region methods

The RL Problem



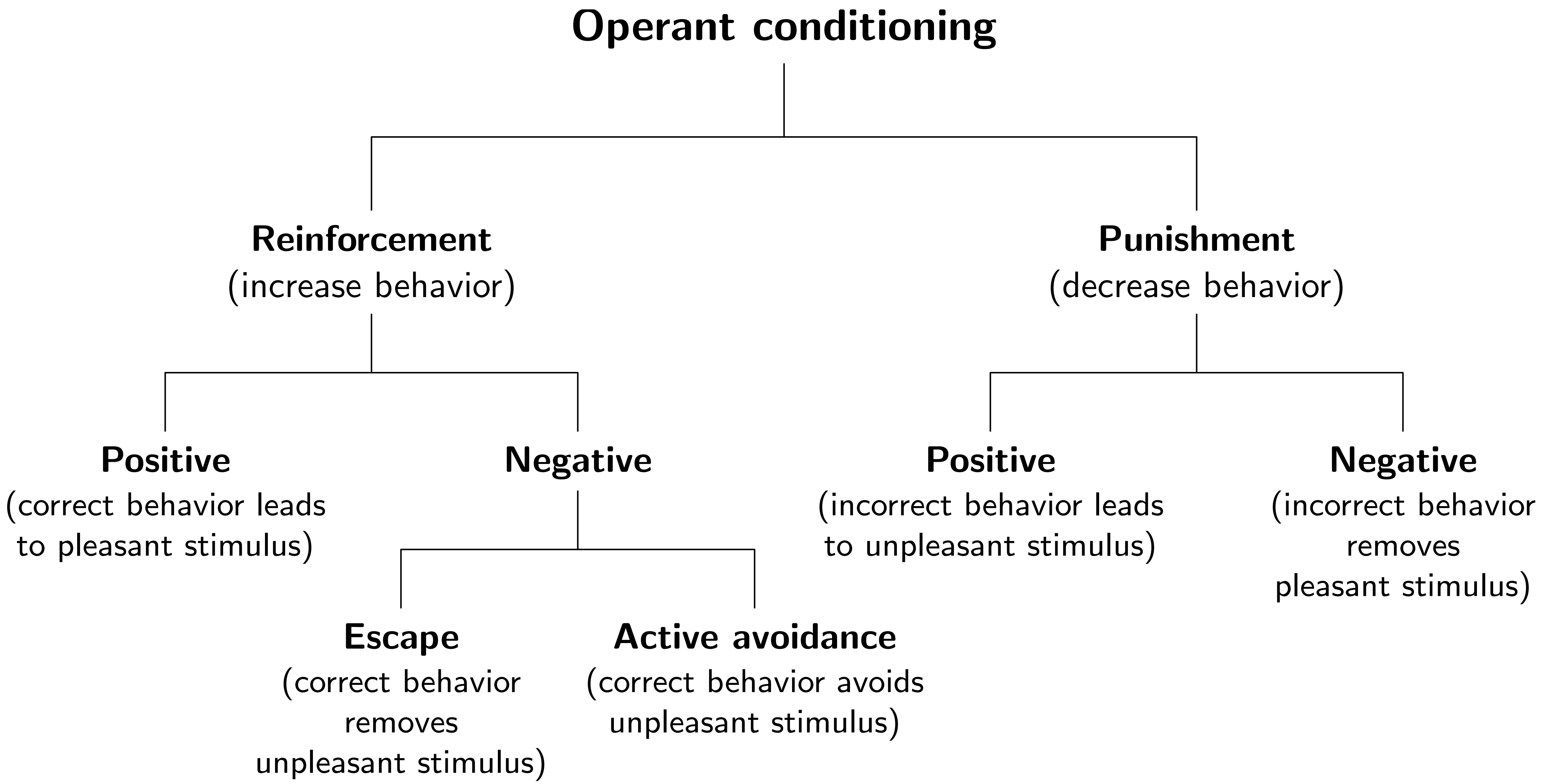
The RL Problem

- Ingredients for success:
    - You learned as you played the game
    - You experimented the different actions
    - As soon as you figured out the goal of the game, you stopped experimenting
    - You used the feedback you got (n. of steps) to figure out the goal of the game
    - When pursuing the goal, you had to think ahead to select the actions

The RL Problem

What is RL?

- Inspired on theory of operant conditioning



Operant conditioning

Reinforcement
(increase behavior)

Positive
(correct behavior leads to pleasant stimulus)

Negative

Escape
(correct behavior removes unpleasant stimulus)

Active avoidance
(correct behavior avoids unpleasant stimulus)

Punishment
(decrease behavior)

Positive
(incorrect behavior leads to unpleasant stimulus)

Negative
(incorrect behavior removes pleasant stimulus)

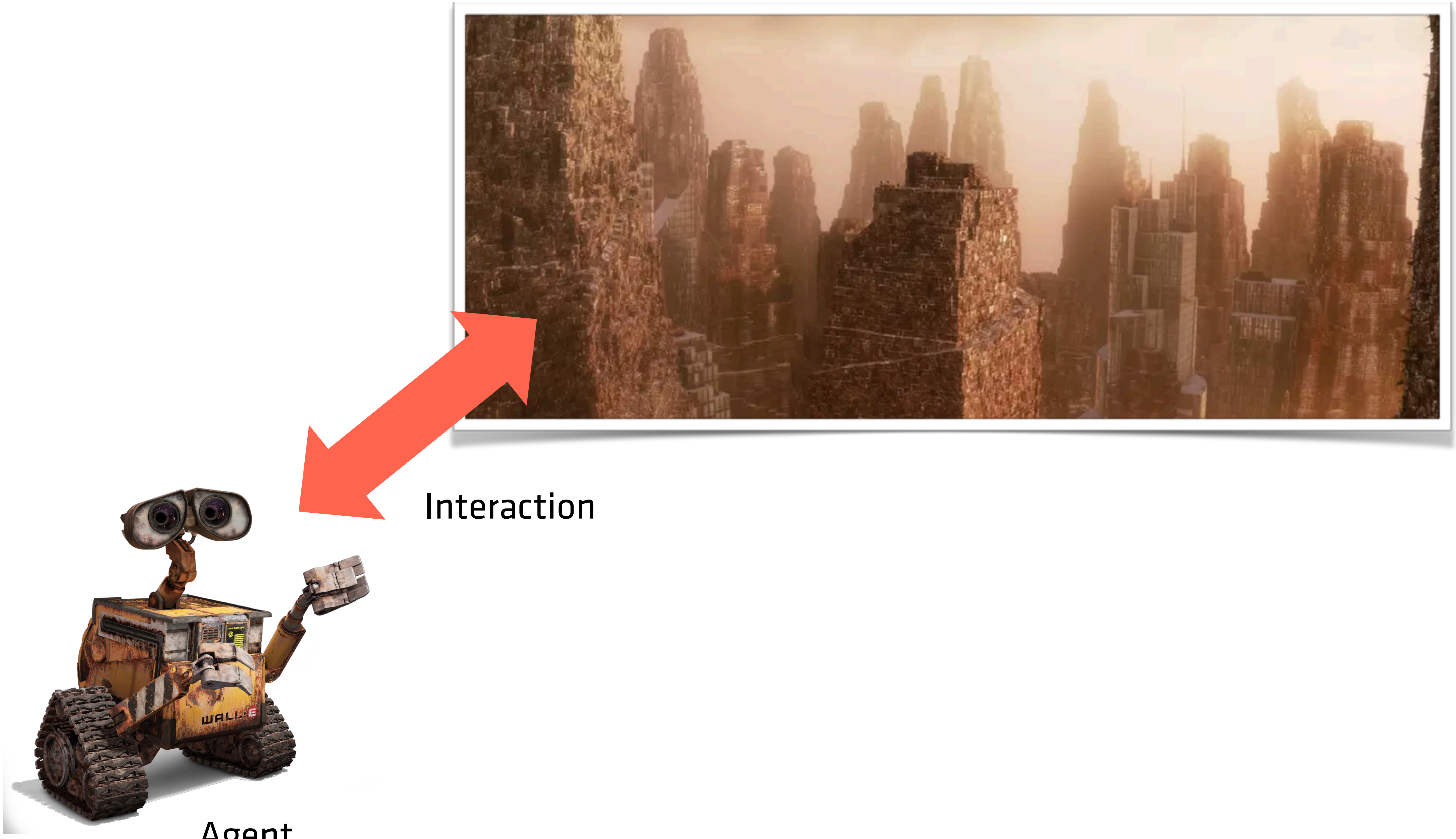
What is RL?

- Computational “counterpart” to operant conditioning
- Class of problems and algorithms to solve those problems
- Learning takes place through the interaction between agent and environment (learning by trial-and-error)
- Learning driven by a “reinforcement signal” rather than examples

Elements in RL

- Key elements in RL:
  - Interactive learning
  - Learning from evaluative feedback
  - Tradeoff between exploration and exploitation
  - Actions impact the future (temporal credit assignment)

Interactive learning

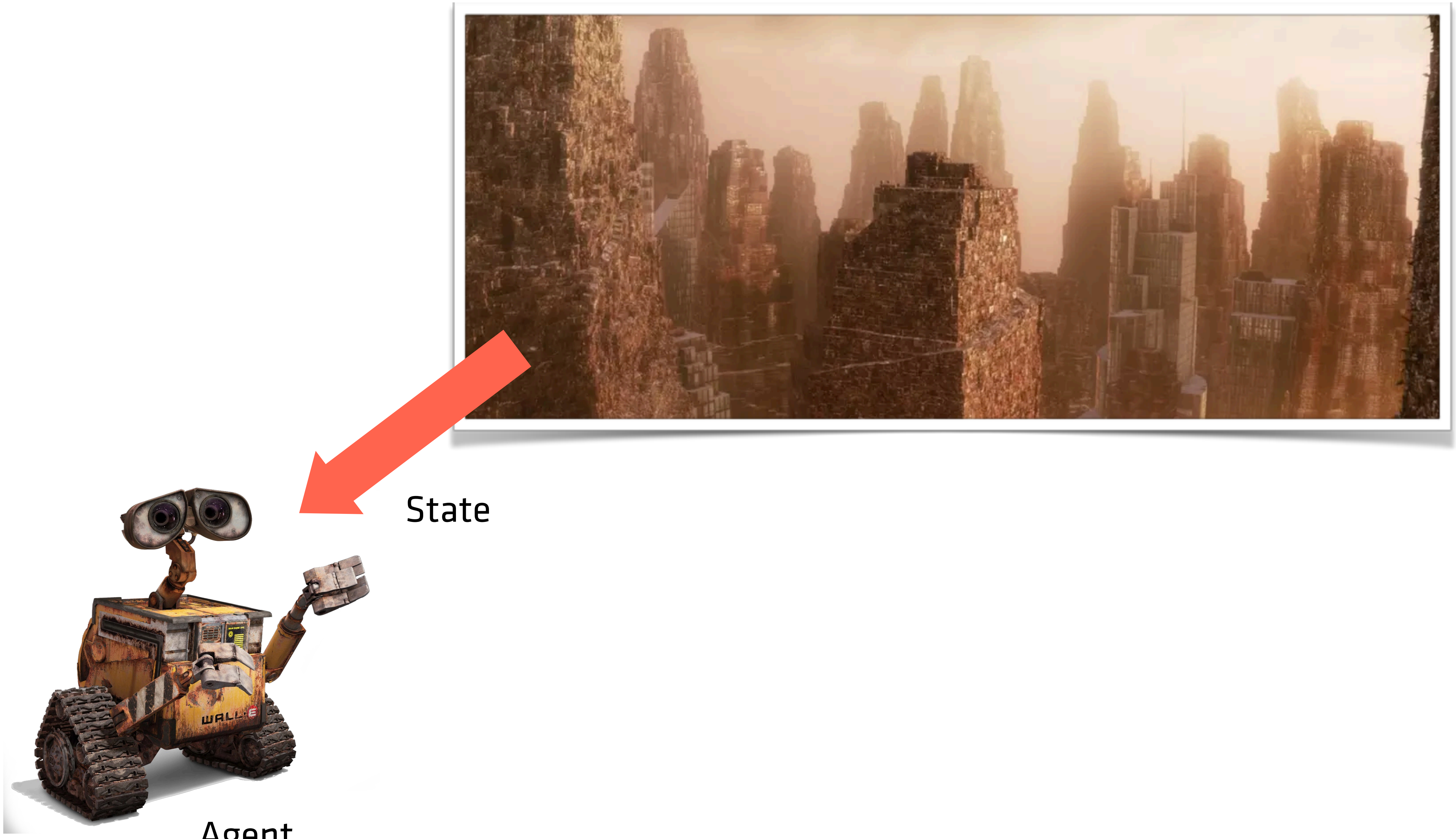


Environment

Interaction

Agent

Interactive learning



Environment

State

Agent

Interactive learning

Environment

Action

Agent

Interactive learning

Environment may change state

Reward

Agent

Markov decision process

- Formalizing the reinforcement learning problem:
    - The state of the world/environment at step t is denoted as S_t
    - The state takes values in some set $\mathcal{S}$ (the state space)

Markov decision process

- Formalizing the reinforcement learning problem:
  - The action of the agent at step t is denoted as A_t
  - The action takes values in some set $\mathcal{A}$ (the action space)

Markov decision process

- Formalizing the reinforcement learning problem:
    - Upon performing an action at time step *t*, the agent gets a (random) reward R_t
    - The reward depends on the state S_t and action A_t as

$$\mathbb{E}\left[R_t\right] = r(S_t, A_t)$$

    - We call *r* the reward function

Markov decision process

- Formalizing the reinforcement learning problem:
    - As a result of the agent's action at time step *t*, the state of the environment at time step $t + 1$ may change
    - We assume that the evolution of the state verifies the Markov property:

$$\mathbb{P}\left[S_{t+1} = s \mid \boxed{\boldsymbol{S}_{0:t} = \boldsymbol{s}_{0:t}, \boldsymbol{A}_{0:t} = \boldsymbol{a}_{0:t}}\right] = \mathbb{P}\left[S_{t+1} = s' \mid \boxed{S_t = s_t, A_t = a_t}\right]$$

Knowledge of the past...

... is subsumed in the present

Markov decision process

- Formalizing the reinforcement learning problem:
  - As a result of the agent's action at time step t, the state of the environment at time step $t + 1$ may change
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$$\mathbb{P}\left[S_{t+1} = s \mid \boldsymbol{S}_{0:t} = \boldsymbol{s}_{0:t}, \boldsymbol{A}_{0:t} = \boldsymbol{a}_{0:t}\right] = \mathbb{P}\left[S_{t+1} = s' \mid S_t = s_t, A_t = a_t\right]$$

  - We call these the transition probabilities, and write

$$\mathbf{P}(s' \mid s, a) = \mathbb{P}\left[S_{t+1} = s' \mid S_t = s, A_t = a\right]$$

Markov decision process

- A Markov decision process is defined as a tuple $(\mathcal{S}, \mathcal{A}, \{\mathbf{P}_a, a \in \mathcal{A}\}, r)$
  - $\boldsymbol{\mathcal{S}}$ is the state space
  - $\boldsymbol{\mathcal{A}}$ is the action space
  - For each action $a \in \boldsymbol{\mathcal{A}}$, $\mathbf{P}_a$ is a matrix with entry ss' given by $\mathbf{P}(s' \mid s,\ a)$
  - r is the reward function

... so what?

Optimality

- A Markov decision process is not actually a problem
  - Provides a mere descriptive model for RL problems
  - What does it mean to solve a model??

Objective

Optimality

- We thus formulate a Markov decision problem (MDP) as follows:

Given a Markov decision process and a function

$$J(\{R_t, t = 0, \dots, \})$$

how can we select the actions $\{A_t\}$ to maximize J?

Policies

- MDPs are formulated in terms of action selection
- A policy is an "action selection rule":
- Define the history at time step *t* as

$$H_t = \{s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{t-1}, a_{t-1}, r_{t-1}, s_t\}$$

  - It is a random variable
  - Depends on the particular action selection

Policies

- A policy is a mapping π between histories and distributions over actions:

$$\pi : \mathcal{H} \to \Delta(\mathcal{A})$$

Set of all
finite histories

Set of distributions
over actions

Policies

- **Types of policies:**
  - Deterministic policies - Each history is mapped to exactly one action

$$\pi : \mathcal{H} \rightarrow \mathcal{A}$$

  - Markov policies - Depend only on the most recent state (may be time-dependent)

$$\pi_{\underset{\nwarrow}{t}} : \mathcal{S} \rightarrow \Delta(\mathcal{A})$$

  - Stationary policies - Depend only on the most recent state (is time-independent)

$$\underset{\nwarrow}{\pi} : \mathcal{S} \rightarrow \Delta(\mathcal{A})$$

Optimality criteria

- J in the previous formulation is the optimality criterion
- There are several possible optimality criteria in the literature
    - Each has advantages and disadvantages
    - The choice should be problem-driven

Optimality criteria

- **(Expected) immediate reward:**

$$J(\{R_t, t = 0, \ldots, \}) = \mathbb{E}\left[R_t\right] = r(S_t, A_t)$$

- Advantages:
    - Simple to optimize:

$$\pi(S_t) = \underset{a \in \mathcal{A}}{\operatorname{argmax}}\, r(S_t, a)$$

- Disadvantages:
    - Only applicable in very specific problems

Optimality criteria

- **(Expected) total reward:**

$$J(\{R_t, t = 0, \ldots, \}) = \mathbb{E}\left[\sum_{t=0}^{\infty} R_t\right]$$

- Advantages:
  - Not myopic
- Disadvantages:
  - Objective not always well-defined (summation may diverge)

Optimality criteria

- **(Expected) average per-step reward:**

$$J(\{R_t, t = 0, \ldots, \}) = \lim_{T\to\infty} \frac{1}{T}\mathbb{E}\left[\sum_{t=0}^{T} R_t\right]$$

- Advantages:
  - Not myopic
  - Independent of initial state of the process
- Disadvantages:
  - Sometimes cumbersome to work with

Optimality criteria

- **(Expected) total discounted reward:**

$$J(\{R_t, t = 0, \ldots, \}) = \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t R_t\right]$$

Discount
$0 \leq \gamma < 1$

- Advantages:
  - Not myopic
  - “Economical” interpretation
- Disadvantages:
  - Depends on the initial state of the process

We henceforth focus on this criterion

Markov decision problem (MDP)

- A Markov decision problem is defined as a tuple $(\mathcal{S}, \mathcal{A}, \{\mathbf{P}_a, a \in \mathcal{A}\}, r, \gamma)$
  - $\boldsymbol{\mathcal{S}}$ is the state space
  - $\boldsymbol{\mathcal{A}}$ is the action space
  - For each action $a \in \boldsymbol{\mathcal{A}}$, $\mathbf{P}_a$ is a matrix with entry ss' given by $\mathbf{P}(s' \mid s,\ a)$
  - r is the reward function
  - γ is the discount

Solving MDPs

Value function

- Let us consider a fixed stationary policy π
  - Action depends only on current state
  - Invariant through time
- In other words,

$$\pi(a \mid s) = \mathbb{P}\left[A_t = a \mid S_t = s\right]$$

Independent of *t*

Value function

- The value of J depends on the initial state
- Let

$$v_\pi(s) = \mathbb{E}_\pi\left[\sum_{t=0}^{\infty} \gamma^t R_t \mid S_0 = s,\right]$$

- $v_\pi(s)$ is the value of J when
  - The agent follows policy π, i.e.,

$$A_t \sim \pi(\cdot \mid S_t)$$

  - The initial state is s

Value function

- The function

$$v_\pi : \mathcal{S} \to \mathbb{R}$$

is called a value function

- It is the value function associated with π
- It verifies the recursive relation

$$v_\pi(s) = \sum_{a \in \mathcal{A}} \pi(a \mid s) \left[r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s,a) v_\pi(s') \right]$$

Immediate
reward

Future total
discounted reward

A computational (parenthesis)

- The relation

$$v_\pi(s) = \sum_{a \in \mathcal{A}} \pi(a \mid s) \left[r(s, a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s, a) v_\pi(s') \right]$$

offers two possibilities to compute v_π

- Solve the associated (linear) system of equations
- Starting with an arbitrary initial estimate $v^{(0)}$, repeatedly go over the update

$$v^{(k+1)}(s) \leftarrow \sum_{a \in \mathcal{A}} \pi(a \mid s) \left[r(s, a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s, a) v^{(k)}(s') \right]$$

A computational (parenthesis)

- The iterative approach with update

$$v^{(k+1)}(s) \leftarrow \sum_{a \in \mathcal{A}} \pi(a \mid s) \left[r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s,a) v^{(k)}(s') \right]$$

  is known as value iteration

- Computing the value function associated with a policy is usually referred as the prediction problem

- It is a dynamic programming approach that, intuitively, “propagates” reward information back through time

... moving on...

Optimal policy

- We say that a policy π^* is optimal if and only if

$$v_{\pi^*}(s) \geq v_\pi(s), \forall \pi, \forall s \in \mathcal{S}$$

- That such a policy exists is a central result in the theory of MDPs

↓

Solving MDP = Computing an optimal policy

Value function 2.0

- The value function for the (an) optimal policy is simply denoted as v^*
- It verifies the recursive relation

$$v^*(s) = \max_{a \in \mathcal{A}} \left[r(s, a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s, a) v^*(s') \right]$$

- The optimal policy can be computed from v^* as

$$\pi^*(s) = \operatorname*{argmax}_{a \in \mathcal{A}} \left[r(s, a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s, a) v^*(s') \right]$$

A computational (parenthesis) 2.0

- The relation

$$v^*(s) = \max_{a \in \mathcal{A}} \left[r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s,a) v^*(s') \right]$$

also offers a possibility to compute v^*

- Starting with an arbitrary initial estimate $v^{(0)}$, repeatedly go over the update

$$v^{(k+1)}(s) \leftarrow \max_{a \in \mathcal{A}} \left[r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s,a) v^{(k)}(s') \right]$$

- An MDP can thus be solved by computing v^* (and π^* from it)

...

Value function 3.0

- **Other useful value functions to be considered**
  - Action-value function (or Q-function) associated with a policy:

$$v_\pi(s) = \sum_{a \in \mathcal{A}} \pi(a \mid s) \underbrace{\boxed{\left[r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s,a) v_\pi(s') \right]}}_{q_\pi(s,a)}$$

Value function 3.0

- **Other useful value functions to be considered**
    - Action-value function (or Q-function) associated with a policy:

$$q_\pi(s,a) = r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s,a) v_\pi(s')$$

    - It verifies the recursive relation

$$q_\pi(s,a) = r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s,a) \sum_{a' \in \mathcal{A}} \pi(a' \mid s') q_\pi(s',a')$$

Value function 3.0

- **Other useful value functions to be considered**
  - Optimal action-value function (or Q-function):

$$v^*(s) = \max_{a \in \mathcal{A}} \underbrace{\left[r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s,a) v^*(s') \right]}_{q^*(s,a)}$$

Value function 3.0

- **Other useful value functions to be considered**
  - Optimal action-value function (or Q-function):

$$q^*(s,a) = r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s,a) v^*(s')$$

  - It verifies the recursive relation

$$q^*(s,a) = r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s,a) \max_{a' \in \mathcal{A}} q^*(s',a')$$

  - Moreover,

$$\pi^*(s) = \operatorname*{argmax}_{a \in \mathcal{A}} q^*(s,a)$$

...

- We can compute q_π and q^* using similar iterative approaches

$$q^{(k+1)}(s, a) \leftarrow r(s, a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s, a) \max_{a' \in \mathcal{A}} q^{(k)}(s', a')$$

$$q^{(k+1)}(s, a) \leftarrow r(s, a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s, a) \sum_{a' \in \mathcal{A}} \pi(a' \mid s') q^{(k)}(s', a')$$

  which are all collectively known as value iteration

- Computing the optimal Q-function is usually referred as the control problem

Value function 3.0

- **Other useful value functions to be considered**
  - Advantage function associated with a policy:

$$\mathrm{adv}_\pi(s, a) = q_\pi(s, a) - v_\pi(s)$$

  - The advantage function does not verify a recursive relation

Wrap up

Key players in RL

- **Immediate reward**
  - Translates the goal of the agent
  - Instantaneous / myopic
- **Policy**
  - Action selection rule
  - Solving an MDP consists in finding the optimal policy

Key players in RL

- **Value function**
  - “Secondary” reward
  - Long-term evaluation of the states
  - Can be used to compute the policy
- **Model (Markov decision process)**
  - Description of the dynamics of the process (transition probabilities)

Solving RL

- Solving an RL problem consists of solving the associated MDP
- Solving an MDP consists of computing the optimal policy.
- E.g.,
    - Use value iteration to compute v^*

or

- Use value iteration to compute q^*
- Use any of the above to compute π^*

Outline of the lecture

- **Part I: RL Primer**
    - The RL Problem
    - Markov Decision Process - A Model for RL Problems
    - Optimality & Dynamic Programming
    - Monte Carlo Approaches
    - Temporal Difference Learning
    - The Policy Gradient Theorem

Reinforcement learning

Reinforcement learning

- Interaction between the agent and the environment
  - Agent observes that $S_t = s$
  - Agent performs an action $A_t = a$
  - Agent gets a reward R_t
  - At the next time step, agent observes $S_{t+1} = s$'
  - ...

Reinforcement learning

- At each step, the agent collects a sample, consisting of a tuple

$$(s, a, r, s')$$

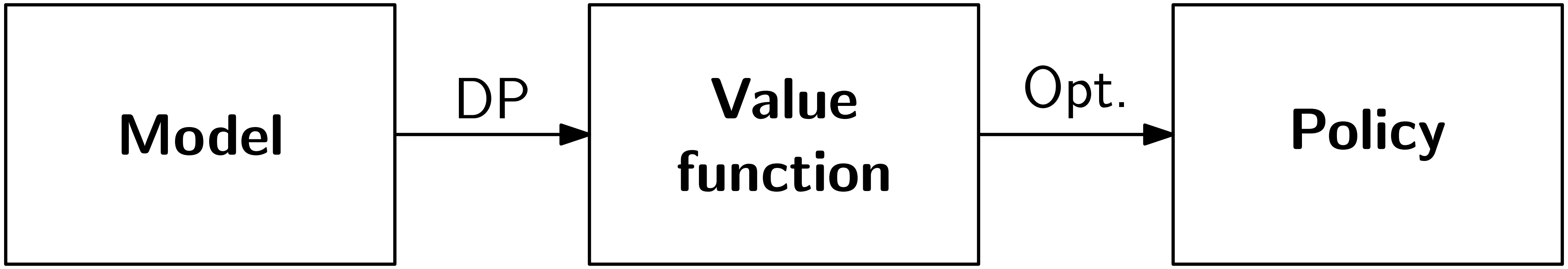
- Each such sample includes information about:
    - The reward, in the triplet $(s,\ a,\ r)$
    - The dynamics, in the triplet $(s,\ a,\ s\text{'})$

Reinforcement learning

- We consider explicitly the two subproblems within RL:
  - The prediction problem (given a policy, compute v_π)
  - The control problem (compute q^*)

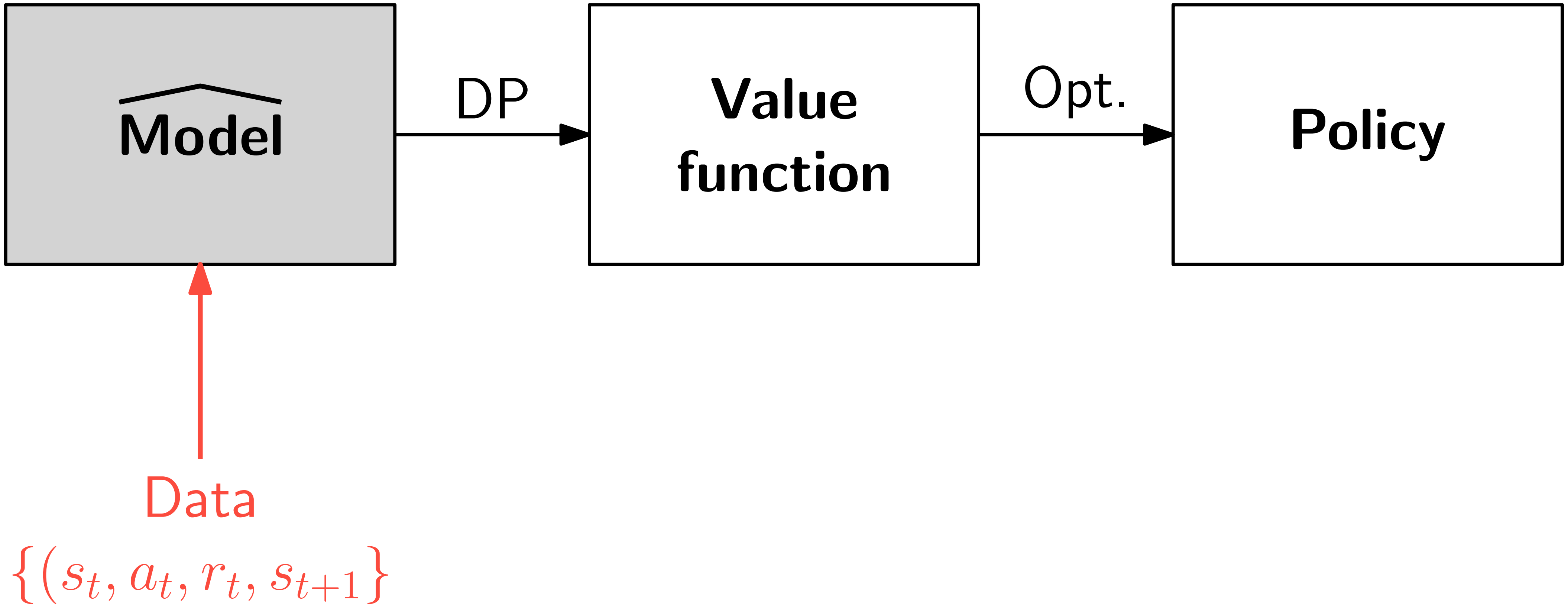
Taxonomy of RL methods

- Solving an MDP:



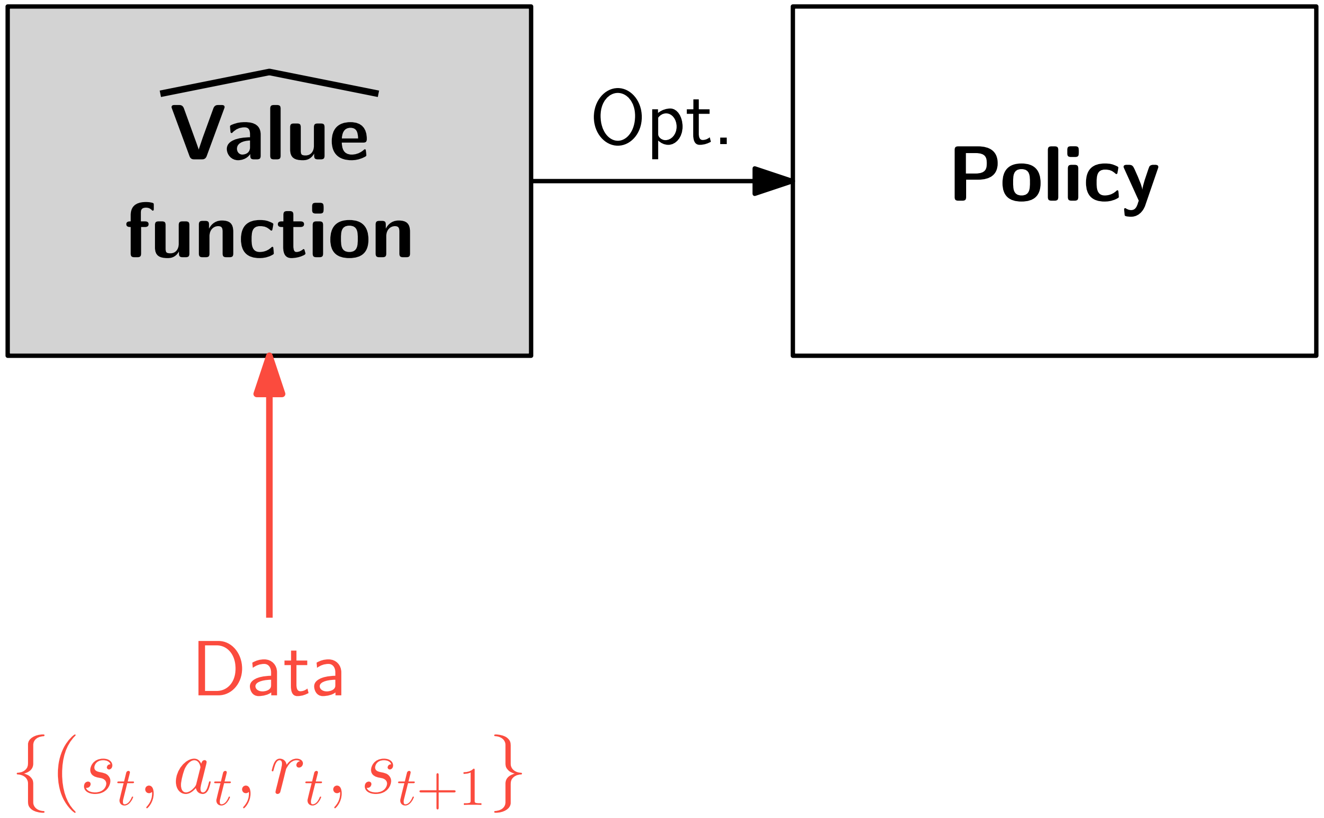
Taxonomy of RL methods

- Model-based methods:



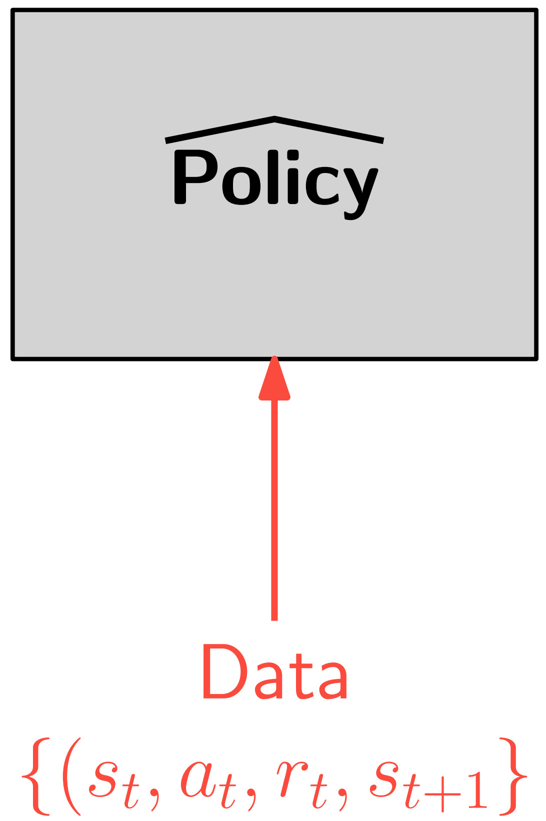
Taxonomy of RL methods

- Value-based methods:



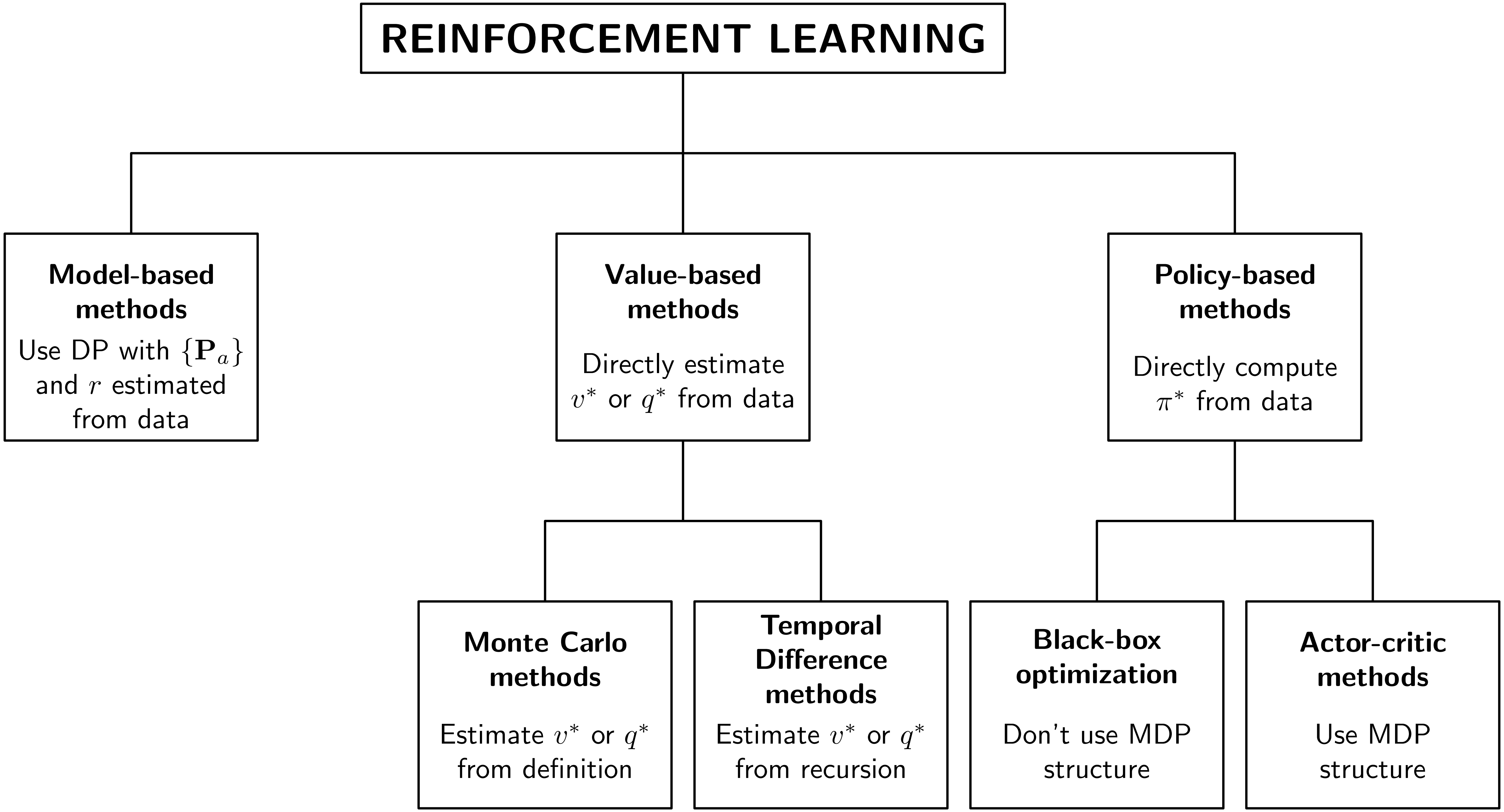
Taxonomy of RL methods

- Policy-based methods:



$$\{(s_t, a_t, r_t, s_{t+1}\}$$

Taxonomy of RL methods



REINFORCEMENT LEARNING

Model-based methods

Use DP with $\{\mathbf{P}_a\}$ and r estimated from data

Value-based methods

Directly estimate v^* or q^* from data

Policy-based methods

Directly compute π^* from data

Monte Carlo methods

Estimate v^* or q^* from definition

Temporal Difference methods

Estimate v^* or q^* from recursion

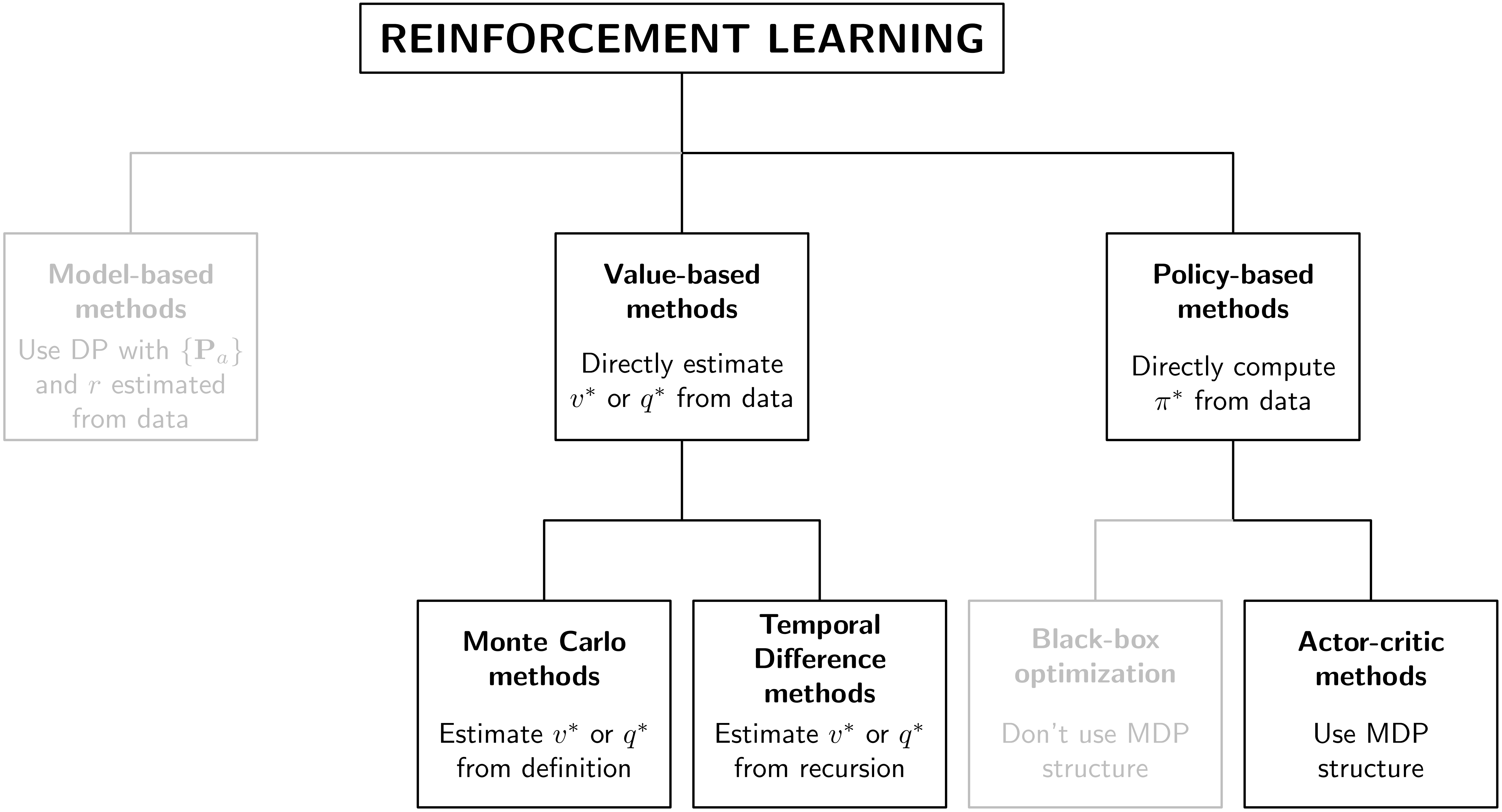
Black-box optimization

Don't use MDP structure

Actor-critic methods

Use MDP structure

Taxonomy of RL methods



REINFORCEMENT LEARNING

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Use DP with $\{\mathbf{P}_a\}$ and r estimated from data

Value-based methods

Directly estimate v^* or q^* from data

Policy-based methods

Directly compute π^* from data

Monte Carlo methods

Estimate v^* or q^* from definition

Temporal Difference methods

Estimate v^* or q^* from recursion

Black-box optimization

Don't use MDP structure

Actor-critic methods

Use MDP structure

Monte Carlo approaches

The prediction problem

- We want to estimate v_π

- We are given a trajectory

$$\mathcal{T} = \{s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{T-1}, a_{T-1}, r_{T-1}, s_T\}$$

obtained while following policy π

- We define the return at time step *t* as

$$G_0 = \sum_{t=0}^{T-1} \gamma^t r_t$$

Using the return

- From the definition of v_π,

$$v_\pi(s_0) \approx \mathbb{E}\left[G_0\right]$$

- Then, given *N* trajectories with a common initial state s_0, we can compute

$$\hat{v}(s_0) = \frac{1}{N}\sum_{n=1}^{N} G_{0,n}$$

or, incrementally,

$$\hat{v}(s_0) \leftarrow \hat{v}(s_0) + \frac{1}{N}(\underset{\uparrow}{G_{0,N}} - \hat{v}(s_0))$$

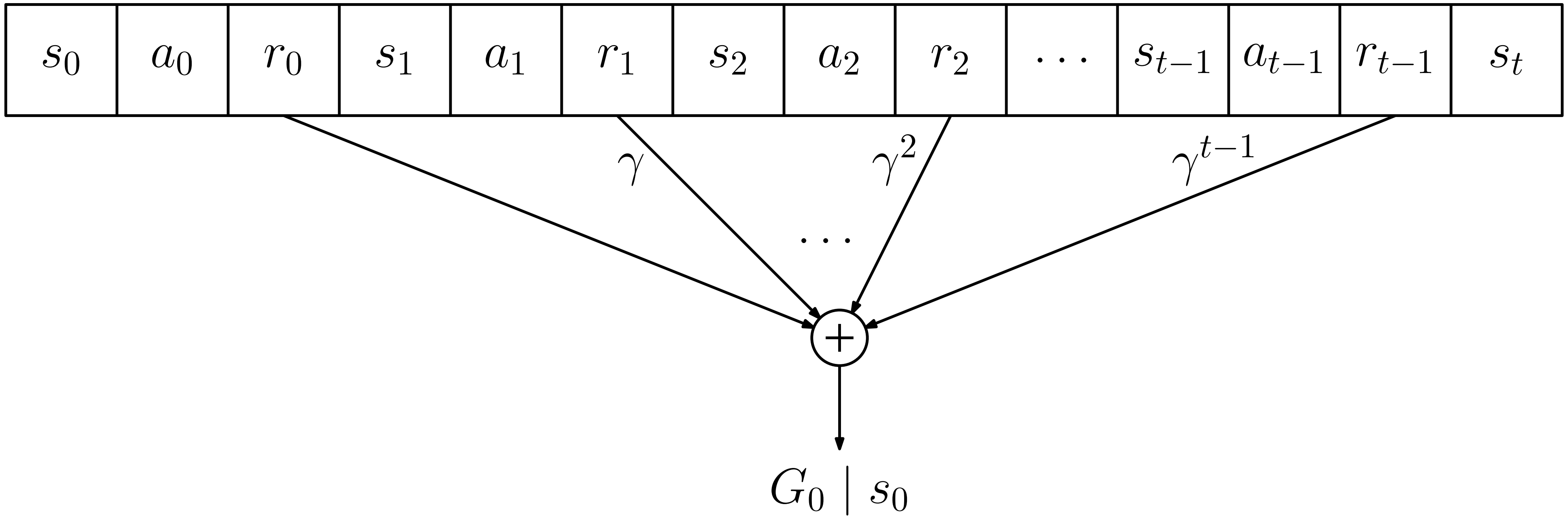
Return for trajectory *N*

Some considerations

- A trajectory

$$\mathcal{T} = \{s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{T-1}, a_{T-1}, r_{T-1}, s_T\}$$

provides returns for multiple states



Some considerations

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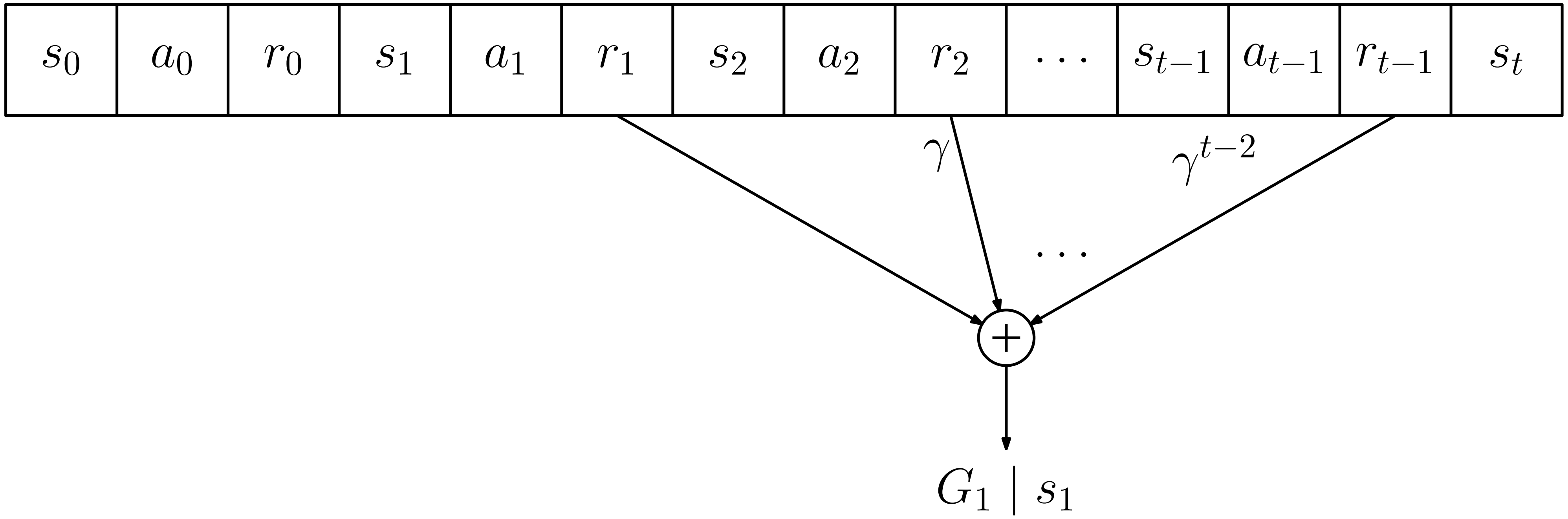
provides returns for multiple states



Some considerations

- A trajectory

$$\mathcal{T} = \{s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{T-1}, a_{T-1}, r_{T-1}, s_T\}$$

  provides returns for multiple states

- Trajectories should visit all states a large number of times

The control problem

- We want to estimate q^*

- We are given a trajectory

$$\mathcal{T} = \{s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{T-1}, a_{T-1}, r_{T-1}, s_T\}$$

obtained by selecting a random action a_0 and following a policy $\pi^{(0)}$ thereafter

Using the return

- From the definition of q_π,

$$q_\pi(s_0, a_0) \approx \mathbb{E}\left[G_0\right]$$

- Then, given *N* trajectories with a common initial state s_0 and initial action a_0, we can compute

$$\hat{q}_\pi(s_0, a_0) = \frac{1}{N}\sum_{n=1}^{N} G_{0,n}$$

or, incrementally,

$$\hat{q}(s_0, a_0) \leftarrow \hat{q}(s_0, a_0) + \frac{1}{N}(G_{0,N} - \hat{q}(s_0, a_0))$$

Some considerations

- To estimate the Q-values for all state-action pairs, we need a large number of trajectories starting in each state-action pair

- To compute the optimal Q-values,

    - Start with arbitrary policy $\pi^{(0)}$ and set $k = 0$

    - Generate multiple trajectories, and estimate $q_{\pi^{(k)}}$

    - Compute policy

Improved policy

$$\pi^{(k+1)}(s) = \underset{a \in \mathcal{A}}{\operatorname{argmax}}\, q_{\pi^{(k)}}(s, a), \forall s$$

- Set $k = k + 1$ and repeat

Temporal difference learning

The prediction problem

- We want to estimate v_π
- We are given a trajectory

$$\mathcal{T} = \{s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{T-1}, a_{T-1}, r_{T-1}, s_T\}$$

obtained while following policy π

The prediction problem

- We know that

$$v_\pi(s) = \sum_{a \in \mathcal{A}} \pi(a \mid s) \left[r(s, a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s, a) v_\pi(s') \right]$$

or, equivalently,

$$v_\pi(s) = \boxed{\mathbb{E}_{A_t \sim \pi(S_t)} \left[R_t + \gamma v_\pi(S_{t+1}) \mid S_t = s \right]}$$

Expectation

The prediction problem

- We know that

$$v_\pi(s) = \sum_{a \in \mathcal{A}} \pi(a \mid s) \left[r(s, a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s, a) v_\pi(s') \right]$$

or, equivalently,

$$v_\pi(s) = \mathbb{E}_{A_t \sim \pi(S_t)} \left[R_t + \gamma v_\pi(S_{t+1}) \mid S_t = s \right]$$

- The value function v_π can be computed iteratively via value iteration using the update

$$v^{(k+1)}(s) \leftarrow \sum_{a \in \mathcal{A}} \pi(a \mid s) \left[r(s, a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s, a) v^{(k)}(s') \right]$$

The prediction problem

- We know that

$$v_\pi(s) = \sum_{a \in \mathcal{A}} \pi(a \mid s) \left[r(s, a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s, a) v_\pi(s') \right]$$

or, equivalently,

$$v_\pi(s) = \mathbb{E}_{A_t \sim \pi(S_t)} \left[R_t + \gamma v_\pi(S_{t+1}) \mid S_t = s \right]$$

- The value function v_π can be computed iteratively via value iteration using the update

$$v^{(k+1)}(s) \leftarrow \mathbb{E}_{A_t \sim \pi(S_t)} \left[R_t + \gamma v^{(k)}(S_{t+1}) \mid S_t = s \right]$$

The prediction problem

- We can approximate the update

$$v^{(k+1)}(s) \leftarrow \mathbb{E}_{A_t \sim \pi(S_t)} \left[R_t + \gamma v^{(k)}(S_{t+1}) \mid S_t = s \right]$$

from samples $\{(s, r_n, s'_n)\}$ as

$$v^{(k+1)}(s) \leftarrow \frac{1}{N} \sum_{n=1}^{N} \left(r_n + \gamma v^{(k)}(s'_n) \right)$$

or, incrementally,

$$v^{(k+1)}(s) \leftarrow v^{(k)}(s) + \frac{1}{N} \left(r_n + \gamma v^{(k)}(s'_n) - v^{(k)}(s) \right)$$

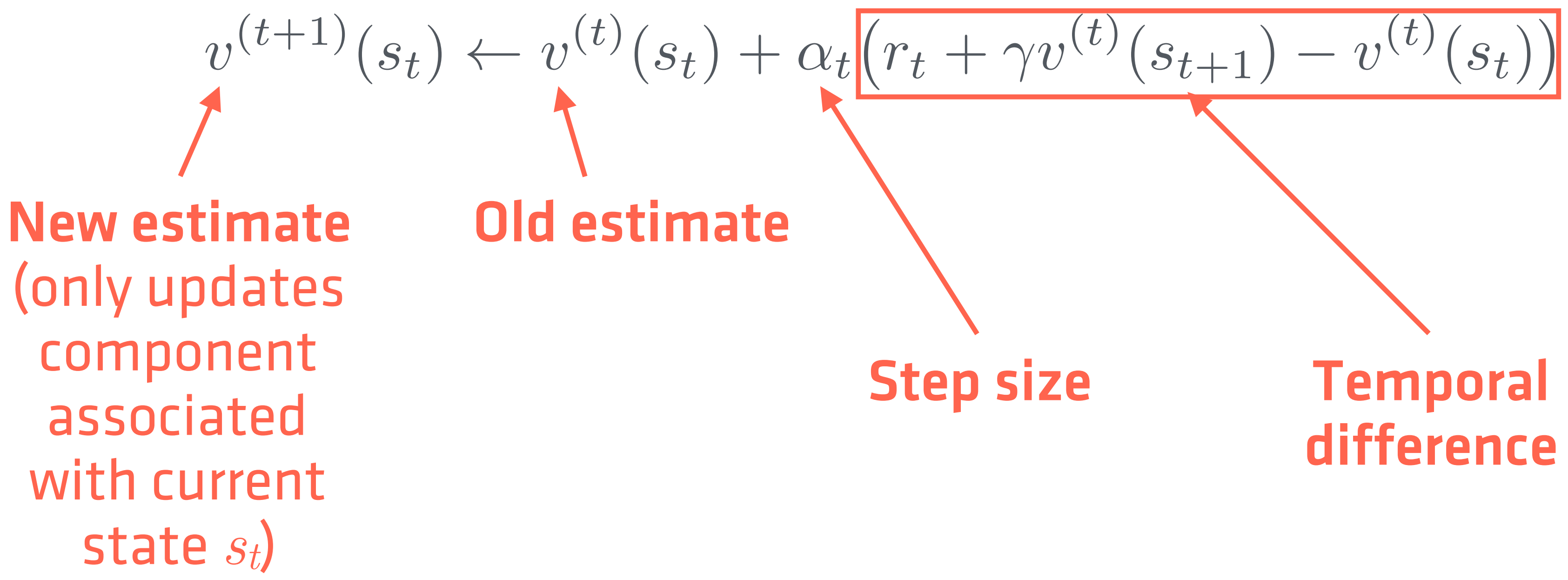
Let’s turn this into a proper algorithm

TD(0)

- Given a (potentially infinite) trajectory

$$\mathcal{T} = \{s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{t-1}, a_{t-1}, r_{t-1}, s_t, \ldots\}$$

generated using policy π, and given an initial estimate $v^{(0)}$ for v_π, TD(0) performs, at each step *t*, the update



$$v^{(t+1)}(s_t) \leftarrow v^{(t)}(s_t) + \alpha_t \boxed{\left(r_t + \gamma v^{(t)}(s_{t+1}) - v^{(t)}(s_t)\right)}$$

New estimate
(only updates component associated with current state s_t)

Old estimate

Step size

Temporal difference

TD(0)

- Given a (potentially infinite) trajectory

$$\mathcal{T} = \{s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{t-1}, a_{t-1}, r_{t-1}, s_t, \ldots\}$$

generated using policy π, and given an initial estimate $v^{(0)}$ for v_π, TD(0) performs, at each step *t*, the update

$$v^{(t+1)}(s_t) \leftarrow v^{(t)}(s_t) + \alpha_t\big(r_t + \gamma v^{(t)}(s_{t+1}) - v^{(t)}(s_t)\big)$$

Compare with what we had

$$v^{(k+1)}(s) \leftarrow v^{(k)}(s) + \frac{1}{N}\big(r_n + \gamma v^{(k)}(s'_n) - v^{(k)}(s)\big)$$

The control problem

- We want to estimate q^*
- We start with the idea used in MC methods (compute q_π, improve π, repeat)
- We are given a trajectory

$$\mathcal{T} = \{s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{T-1}, a_{T-1}, r_{T-1}, s_T\}$$

obtained while following some initial policy π

The control problem

- Repeating the same reasoning,

$$q_\pi(s, a) = \mathbb{E}_{A_{t+1} \sim \pi(S_{t+1})} [R_t + \gamma q_\pi(S_{t+1}, A_{t+1}) \mid S_t = s, A_t = a]$$

leading to the update

$$q^{(k+1)}(s, a) \leftarrow \mathbb{E}_{A_{t+1} \sim \pi(S_{t+1})} \left[R_t + \gamma q^{(k)}(S_{t+1}, A_{t+1}) \mid S_t = s, A_t = a\right]$$

The control problem

- Then, given a trajectory

$$\mathcal{T} = \{s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{T-1}, a_{T-1}, r_{T-1}, s_T\}$$

generated using a policy π, and given an initial estimate $q^{(0)}$ for q_π, update

$$q^{(t+1)}(s_t, a_t) \leftarrow q^{(t)}(s_t, a_t) + \alpha_t\big(r_t + \gamma q^{(t)}(s_{t+1}, a_{t+1}) - q^{(t)}(s_t, a_t)\big)$$

- After some iterations, compute a new policy

$$\pi(s) \leftarrow \underset{a \in \mathcal{A}}{\operatorname{argmax}}\, q^{(t)}(s, a)$$

SARSA

- This approach runs the following cycle:
  - Start with a policy
  - Evaluate it, computing its associated Q-function
  - Update the policy
  - Repeat
- Each update to $q^{(t)}$ uses a sample $(s_t,\ a_t,\ r_t,\ s_{t+1},\ a_{t+1})$
- The algorithm is thus named SARSA

Can we learn q^* directly?

The control problem

- Let us again repeat the same reasoning

$$q^*(s,a) = \mathbb{E}\left[R_t + \gamma \max_{a \in \mathcal{A}} q^*(S_{t+1}, a) \mid S_t = s, A_t = a\right]$$

we get the update

$$q^{(k+1)}(s,a) \leftarrow \mathbb{E}\left[R_t + \gamma \max_{a \in \mathcal{A}} q^{(k)}(S_{t+1}, a) \mid S_t = s, A_t = a\right]$$

Q-learning

- Then, given a (potentially infinite) trajectory

$$\mathcal{T} = \{s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{t-1}, a_{t-1}, r_{t-1}, s_t, \ldots\}$$

generated using an arbitrary policy π, and given an initial estimate $q^{(0)}$ for q^*, update

$$q^{(t+1)}(s_t, a_t) \leftarrow q^{(t)}(s_t, a_t) + \alpha_t(r_t + \gamma \max_{a \in \mathcal{A}} q^{(t)}(s_{t+1}, a) - q^{(t)}(s_t, a_t))$$

Summarizing...

- TD(0) is used to compute the value function for a given policy
- It relies on the update

$$v^{(t+1)}(s_t) \leftarrow v^{(t)}(s_t) + \alpha_t\big(r_t + \gamma v^{(t)}(s_{t+1}) - v^{(t)}(s_t)\big)$$

Summarizing...

- SARSA and Q-learning are used to compute the optimal Q-function
- SARSA relies on the update

$$q^{(t+1)}(s_t, a_t) \leftarrow q^{(t)}(s_t, a_t) + \alpha_t\big(r_t + \gamma q^{(t)}(s_{t+1}, a_{t+1}) - q^{(t)}(s_t, a_t)\big)$$

- SARSA learns the Q-function for the policy used to obtain the samples

☞ On-policy learning

- In order to compute the optimal policy, it must slowly adjust the policy used to obtain the samples

Summarizing...

- Q-learning relies on the update

$$q^{(t+1)}(s_t, a_t) \leftarrow q^{(t)}(s_t, a_t) + \alpha_t(r_t + \gamma \max_{a \in \mathcal{A}} q^{(t)}(s_{t+1}, a) - q^{(t)}(s_t, a_t))$$

- Q-learning learns the optimal Q-function, independently of the policy used to obtain the samples

☞ Off-policy learning

The policy gradient theorem

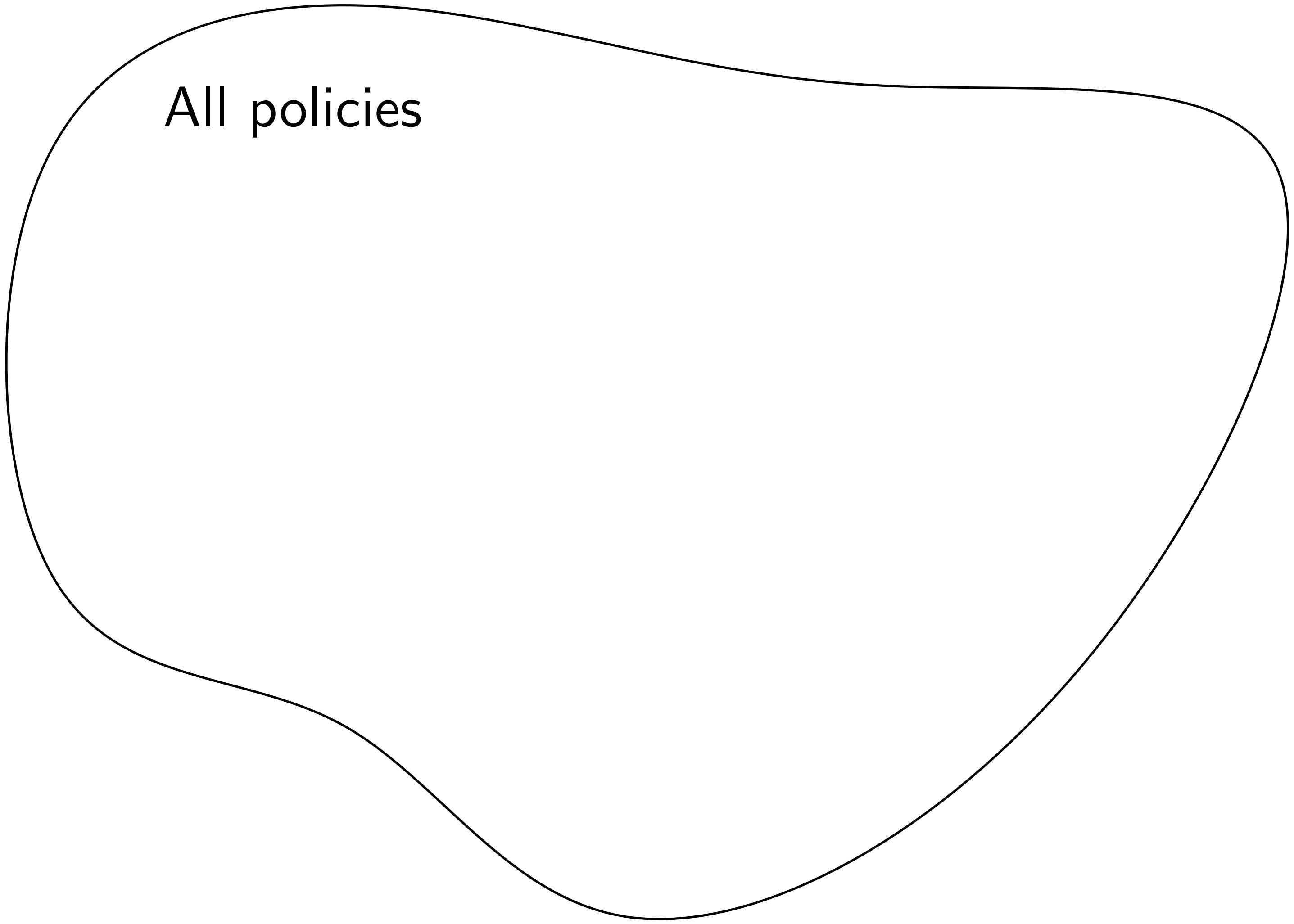
Policy-based methods

- The goal is to compute π^* directly

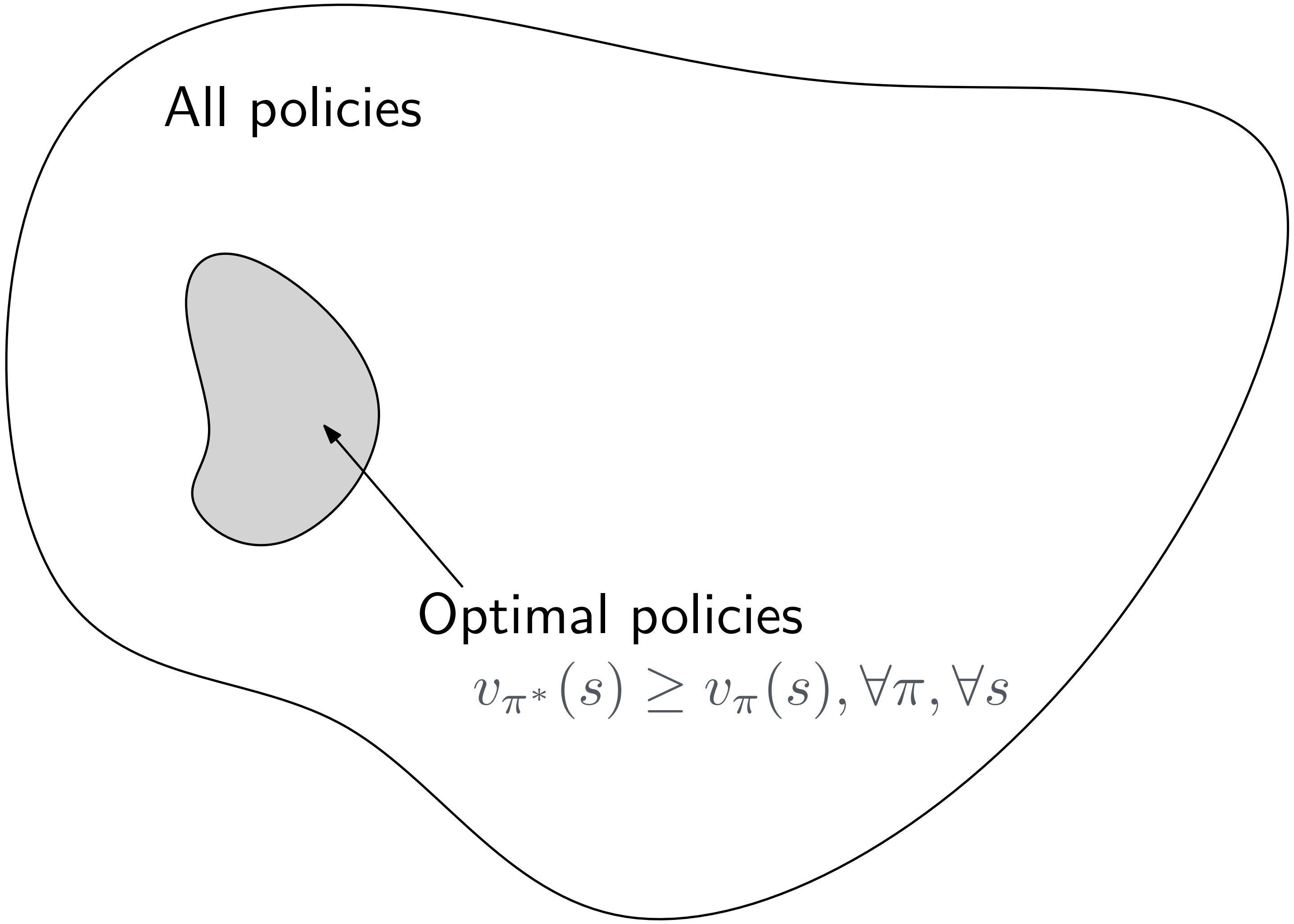
- We depart from a parameterized family of policies, π_θ

... however...

Policy-based methods



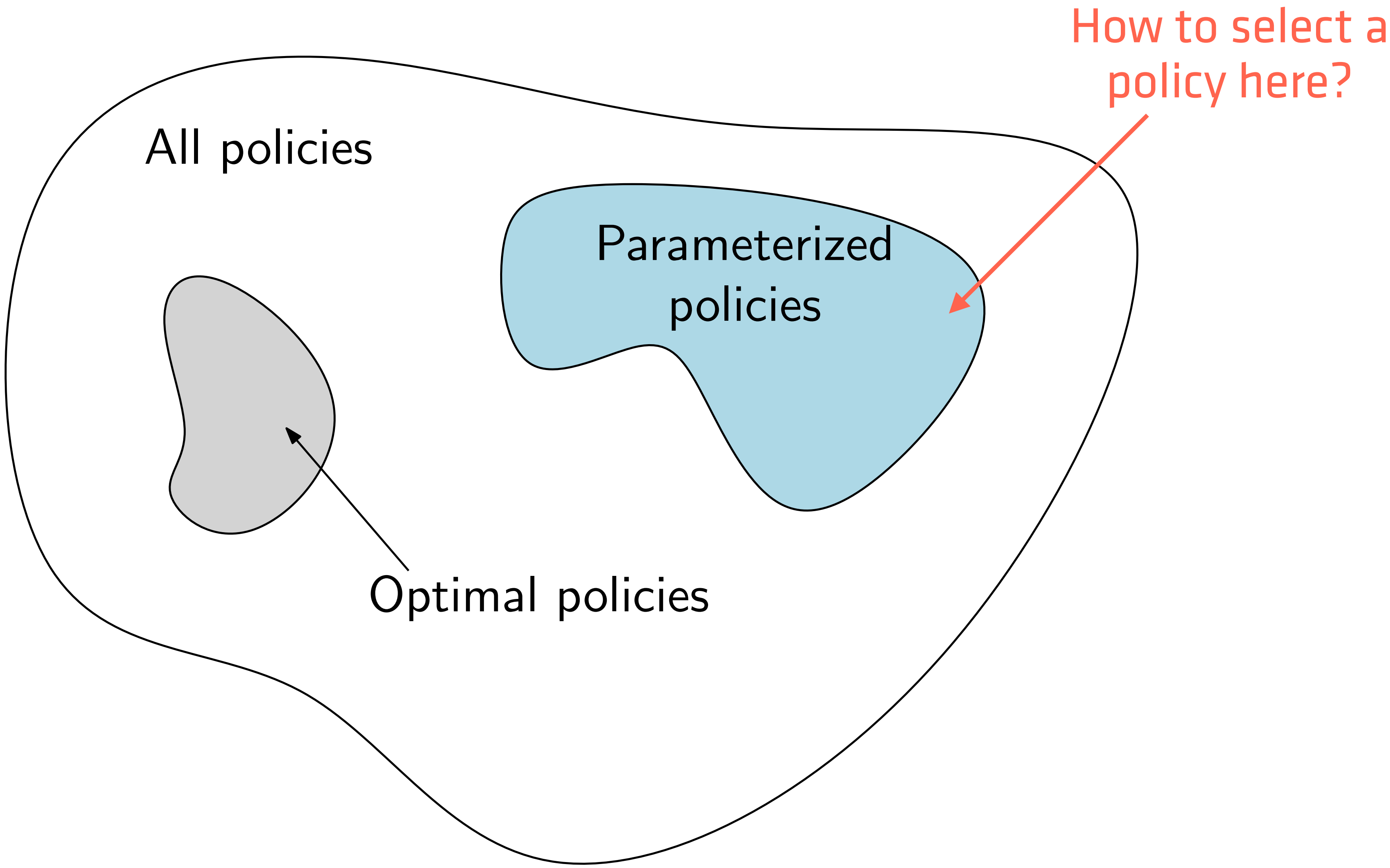
Policy-based methods


All policies

Optimal policies

$v_{\pi^*}(s) \geq v_\pi(s), \forall \pi, \forall s$

Policy-based methods



Revisiting optimality criterion

- When considering the set of all policies, state-wise optimization is possible
- When considering a restricted set of policies, state-wise optimization may not be possible

Revisiting optimality criterion

- Recall that our goal is to maximize

$$J(\{R_t, t = 0, \ldots, \}) = \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t R_t\right]$$

- We consider that the initial state of the MDP follows some initial distribution μ

- To explicitly indicate the dependence of J on the initial distribution μ and the policy π used to generate $\{R_t,\ t = 1,\ \ldots\}$, we write

$$J(\pi; \mu) \triangleq \mathbb{E}_\pi\left[\sum_{t=0}^{\infty} \gamma^t R_t \mid S_0 \sim \mu\right]$$

Interesting relations

- We have that
  - $v_\pi(s) = J(\pi; \mu)$ when $\mu(s') = \mathbb{I}(s' = s)$
  - Conversely, for an arbitrary distribution μ,

$$J(\pi; \mu) = \sum_{s \in \mathcal{S}} \mu(s) v_\pi(s)$$

RL using gradient ascent

- We can now optimize J with respect to the parameters of the policy

- Using gradient ascent, we get an algorithm

$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha \nabla_{\theta} J(\pi_{\theta}; \mu)$$

**Methods based on this idea
are globally called
“policy-gradient methods”**

Policy gradient

- We now compute the policy gradient

$$\begin{aligned} \nabla_\theta J(\pi_\theta;\mu) &= \nabla_\theta \sum_{s\in\mathcal{S}} \mu(s) v_{\pi_\theta}(s) \\ &= \sum_{s\in\mathcal{S}} \mu(s) \boxed{\nabla_\theta v_{\pi_\theta}(s)} \end{aligned}$$

Let us consider
this term alone

Policy gradient

- Since

$$v_{\pi_\theta}(s) = \sum_{a \in \mathcal{A}} \pi_\theta(a \mid s) q_{\pi_\theta}(s, a)$$

it holds that

$$\nabla_\theta v_{\pi_\theta}(s) = \sum_{a \in \mathcal{A}} \left[\nabla_\theta \pi_\theta(a \mid s) q_{\pi_\theta}(s, a) + \pi_\theta(a \mid s) \boxed{\nabla_\theta q_{\pi_\theta}(s, a)}\right]$$

We now look at this term

Policy gradient

- Since

$$q_{\pi_\theta}(s, a) = r(s, a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s, a) v_{\pi_\theta}(s')$$

it holds that

$$\nabla_\theta q_{\pi_\theta}(s, a) = \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s, a) \nabla_\theta v_{\pi_\theta}(s')$$

Policy gradient

- Putting everything together,

Factoring this out

$$\nabla_\theta v_{\pi_\theta}(s) = \sum_{a\in\mathcal{A}} \left[\nabla_\theta \pi_\theta(a \mid s) q_{\pi_\theta}(s,a) + \gamma \pi_\theta(a \mid s) \sum_{s'\in\mathcal{S}} \mathbf{P}(s' \mid s,a) \nabla_\theta v_{\pi_\theta}(s') \right]$$

$$= \sum_{a\in\mathcal{A}} \pi_\theta(a \mid s) \left[\frac{\nabla_\theta \pi_\theta(a \mid s)}{\pi_\theta(a \mid s)} q_{\pi_\theta}(s,a) + \gamma \sum_{s'\in\mathcal{S}} \mathbf{P}(s' \mid s,a) \nabla_\theta v_{\pi_\theta}(s') \right]$$

This is just

$\nabla_\theta \log \pi_\theta(a \mid s)$

Policy gradient

- Putting everything together,

$$\begin{aligned}\nabla_\theta v_{\pi_\theta}(s) &= \sum_{a\in\mathcal{A}} \left[\nabla_\theta \pi_\theta(a \mid s) q_{\pi_\theta}(s,a) + \gamma \pi_\theta(a \mid s) \sum_{s'\in\mathcal{S}} \mathbf{P}(s' \mid s,a) \nabla_\theta v_{\pi_\theta}(s')\right] \\ &= \sum_{a\in\mathcal{A}} \pi_\theta(a \mid s) \left[\nabla_\theta \log \pi_\theta(a \mid s) q_{\pi_\theta}(s,a) + \gamma \sum_{s'\in\mathcal{S}} \mathbf{P}(s' \mid s,a) \nabla_\theta v_{\pi_\theta}(s')\right]\end{aligned}$$

- Recursive relation reminiscent of that for v_π

Plays the role
of “reward”

Policy gradient

- Unfolding the recursion finally yields

$$\nabla_\theta J(\pi_\theta; \mu) = \sum_{s \in \mathcal{S}} \mu_\theta(s) \sum_{a \in \mathcal{A}} \pi_\theta(a \mid s) \nabla_\theta \log \pi_\theta(a \mid s) q_{\pi_\theta}(s, a)$$

or, equivalently,

$$\nabla_\theta J(\pi_\theta; \mu) = \mathbb{E}_{S \sim \mu_\theta, A \sim \pi(\cdot \mid S)} \left[\nabla_\theta \log \pi_\theta(A \mid S) q_{\pi_\theta}(S, A) \right]$$

- The distribution μ_θ translates the "discounted visitation frequency" under π_θ

- Can be sampled by sampled the MDP while following π_θ

REINFORCE

- The gradient is just the

- Given a trajectory obtained from π_θ and with initial state sampled from μ_θ,

$$\nabla_\theta J(\pi_\theta; \mu) \approx \sum_{t=0}^{T} \gamma^t G_t \log \pi_\theta(a_t \mid s_t)$$

Estimate of
$q_\pi(s_t, a_t)$

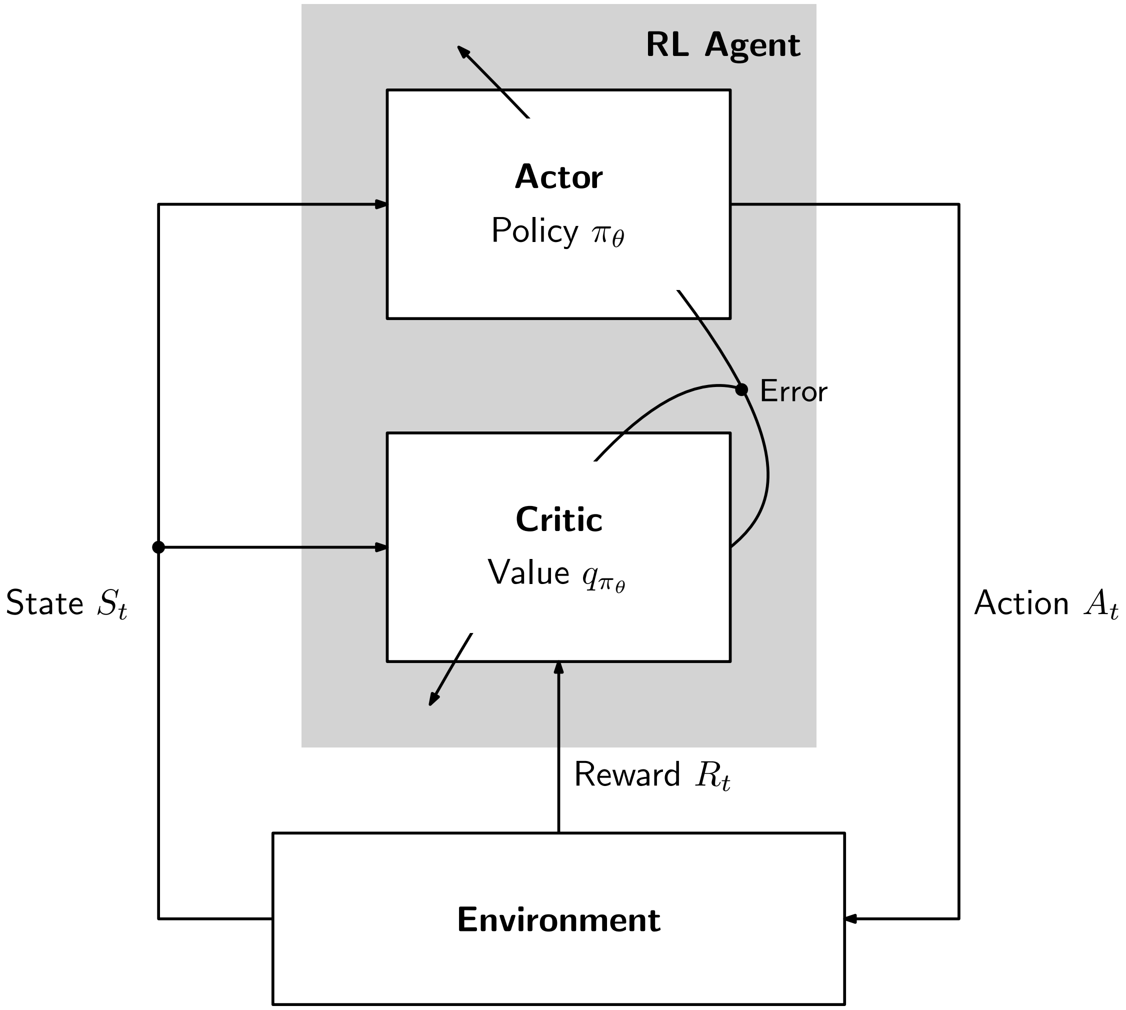
Actor-critic architecture

- To compute the gradient, we require an estimate of the Q-values
- REINFORCE uses a simple Monte Carlo approach to build such estimate
- However, other approaches can be used (e.g., temporal-difference learning)

Actor-critic architecture

- The RL algorithm comprises two components:
  - An actor, responsible for executing the policy π_θ
  - A critic, responsible for evaluating the policy (computing q_π)

Actor-critic architecture



TD-based actor-critic

- For example, we can have an actor-critic based on TD-learning:
  - Given a trajectory

$$\mathcal{T} = \{s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{t-1}, a_{t-1}, r_{t-1}, s_t, \ldots\}$$

  - Update the Q-value estimates as

$$q^{(t+1)}(s_t, a_t) = q^{(t)}(s_t, a_t) + \alpha_t(r_t + \gamma q^{(t)}(s_{t+1}, a_{t+1}) - q^{(t)}(s_t, a_t))$$

  - Update gradient term

$$\boldsymbol{\theta}^{(t+1)} = \boldsymbol{\theta}^{(t)} + \beta_t \gamma^t q^{(t+1)}(s_t, a_t) \nabla_\theta \log \pi_\theta(s_t, a_t)$$

Considerations

- PG/AC architectures are convenient with function approximation
    - Gradient does not depend on q_π but on a projection thereof
- Variations of the gradient (e.g., natural gradient) can also be used:
- Discount is cumbersome to deal with
    - Many PG/AC applications instead adopt the average per-step reward
- Fully incremental approaches suffer from high variance and are seldom used

Adding a baseline

- Consider once again the gradient expression

$$\nabla_\theta J(\pi_\theta;\mu) = \mathbb{E}_{S\sim\mu_\theta, A\sim\pi_\theta(\cdot|S)}\left[\nabla_\theta \log \pi_\theta(A \mid S) q_{\pi_\theta}(S, A)\right]$$

- Gradient estimated from samples

- Estimates plagued by high variance (sensitivity to the particular samples)

Adding a baseline

- Result from theory of Monte Carlo integration:
  - Use of a baseline can often improve variance of sample-based estimates

$$\mathbb{E}\left[f(X)\right] \approx \frac{1}{N}\sum_{n=1}^{N} f(x_n)$$

$$\mathbb{E}\left[f(X) - g(X)\right] \approx \frac{1}{N}\sum_{n=1}^{N}(f(x_n) - g(x_n)) \longrightarrow \text{Less variance}$$

Baseline
($\mathbb{E}\left[g(X)\right]$ known)

Adding a baseline

- Consider an arbitrary function

$$b : \mathcal{S} \to \mathbb{R}$$

- Then,

$$\sum_{a \in \mathcal{A}} \nabla_\theta \pi_\theta(a \mid s) b(s) = \ ?$$

Adding a baseline

- Consider an arbitrary function

$$b : \mathcal{S} \to \mathbb{R}$$

- Then,

$$\sum_{a \in \mathcal{A}} \nabla_\theta \pi_\theta(a \mid s) b(s) = \nabla_\theta \left[\sum_{a \in \mathcal{A}} \pi_\theta(a \mid s) \right] b(s) = 0$$

Adding a baseline

- But then

$$\nabla_\theta J(\pi_\theta; \mu) = \mathbb{E}_{S\sim\mu_\theta, A\sim\pi(\cdot|S)}\left[\nabla_\theta \log \pi_\theta(A \mid S) q_{\pi_\theta}(S, A) - \nabla_\theta \log \pi_\theta(A \mid S) b(S)\right]$$

or, equivalently,

$$\nabla_\theta J(\pi_\theta; \mu) = \mathbb{E}_{S\sim\mu_\theta, A\sim\pi(\cdot|S)}\left[\nabla_\theta \log \pi_\theta(A \mid S)(q_{\pi_\theta}(S, A) - b(S))\right]$$

Best baseline:
$v_{\pi_\theta}(S)$

Adding a baseline

- But then

$$\nabla_\theta J(\pi_\theta; \mu) = \mathbb{E}_{S \sim \mu_\theta, A \sim \pi(\cdot \mid S)} \left[\nabla_\theta \log \pi_\theta(A \mid S) q_{\pi_\theta}(S, A) - \nabla_\theta \log \pi_\theta(A \mid S) b(S) \right]$$

or, equivalently,

$$\nabla_\theta J(\pi_\theta; \mu) = \mathbb{E}_{S \sim \mu_\theta, A \sim \pi(\cdot \mid S)} \left[\nabla_\theta \log \pi_\theta(A \mid S) \boxed{(q_{\pi_\theta}(S, A) - v_{\pi_\theta}(S))} \right]$$

Advantage
$\mathrm{adv}_\pi(S, A)$

Adding a baseline

- But then

$$\nabla_\theta J(\pi_\theta;\mu) = \mathbb{E}_{S\sim\mu_\theta, A\sim\pi(\cdot\mid S)}\left[\nabla_\theta \log \pi_\theta(A\mid S) q_{\pi_\theta}(S,A) - \nabla_\theta \log \pi_\theta(A\mid S) b(S)\right]$$

or, equivalently,

$$\nabla_\theta J(\pi_\theta;\mu) = \mathbb{E}_{S\sim\mu_\theta, A\sim\pi(\cdot\mid S)}\left[\nabla_\theta \log \pi_\theta(A\mid S) \mathrm{adv}_{\pi_\theta}(S,A)\right]$$

☞ This is the underlying form of most current AC algorithms

Outline of the lecture

- **Part I: RL Primer**
    - The RL Problem
    - Markov Decision Process - A Model for RL Problems
    - Optimality & Dynamic Programming
    - Monte Carlo Approaches
    - Temporal Difference Learning
    - The Policy Gradient Theorem

Outline of the lecture

- **Part II: Deep RL**
  - From RL to Deep RL
  - DQN
  - Deep advantage actor-critic methods
  - Trust region methods

RL in large domains

- **Plan:**
    - Revisit temporal difference learning in large domains
    - Revisit policy-gradient methods in large domains

Temporal difference learning revisited

TDL in large domains

- Temporal difference learning methods require explicit updates:

$$v^{(t+1)}(s_t) \leftarrow v^{(t)}(s_t) + \alpha_t\big(r_t + \gamma v^{(t)}(s_{t+1}) - v^{(t)}(s_t)\big)$$

$$q^{(t+1)}(s_t, a_t) \leftarrow q^{(t)}(s_t, a_t) + \alpha_t(r_t + \gamma \max_{a \in \mathcal{A}} q^{(t)}(s_{t+1}, a) - q^{(t)}(s_t, a_t))$$

Component s_t is explicitly updated

Component (s_t, a_t) is explicitly updated

TDL in large domains

- For large domains, function approximation is necessary
  - We can no longer compute v_π or q^* exactly
  - Instead, we consider parameterized families of functions

TDL in large domains

- Example: TD-learning with linear function approximation
    - We consider the family of functions of the form

$$v(s;\boldsymbol{w}) = \boldsymbol{w}^\top \boldsymbol{\phi}(s)$$

where $\boldsymbol{w}$ is a vector of parameters

- We update the parameters $\boldsymbol{w}$ as

$$\boldsymbol{w}^{(t+1)} \leftarrow \boldsymbol{w}^{(t)} + \alpha_t \boldsymbol{\phi}(s_t)(r_t + \gamma v(s_{t+1};\boldsymbol{w}^{(t)}) - v(s_t;\boldsymbol{w}^{(t)}))$$

$\updownarrow$ Compare

$$v^{(t+1)}(s_t) \leftarrow v^{(t)}(s_t) + \alpha_t \big(r_t + \gamma v^{(t)}(s_t) - v^{(t)}(s_t)\big)$$

TDL in large domains

- Another example: Q-learning with linear function approximation
  - We consider the family of functions of the form

$$q(s, a; \boldsymbol{w}) = \boldsymbol{w}^\top \boldsymbol{\phi}(s, a)$$

where $\boldsymbol{w}$ is a vector of parameters

  - We update the parameters $\boldsymbol{w}$ as

$$\boldsymbol{w}^{(t+1)} \leftarrow \boldsymbol{w}^{(t)} + \alpha_t \boldsymbol{\phi}(s_t, a_t)(r_t + \gamma \max_{a \in \mathcal{A}} q(s_{t+1}, a; \boldsymbol{w}^{(t)}) - q(s_t, a_t; \boldsymbol{w}^{(t)}))$$

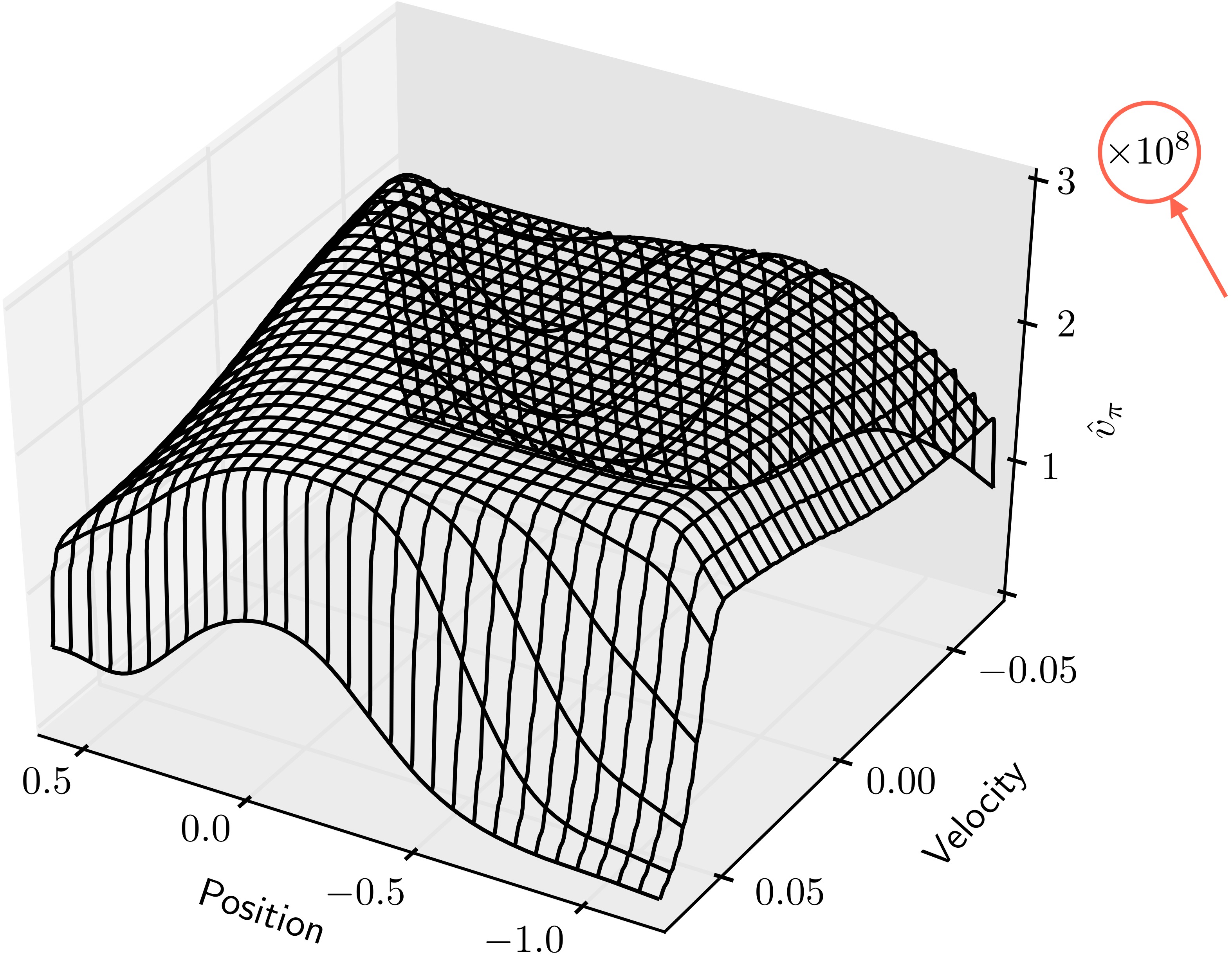
$$\updownarrow$$

Compare

$$q^{(t+1)}(s_t, a_t) \leftarrow q^{(t)}(s_t, a_t) + \alpha_t (r_t + \gamma \max_{a \in \mathcal{A}} q^{(t)}(s_{t+1}, a) - q^{(t)}(s_t, a_t))$$

The problem of function approximation

- Unfortunately, temporal-difference methods may diverge with function approximation



The problem of function approximation

- Issues with function approximation in RL:
  - Bootstrapping - the target is built from current estimate
  - Sample correlation - samples come from a trajectory

Given the previous difficulties, how can we combine ANNs with RL?

Combining ANNs and RL

- We address directly the control problem
- Three ideas:
  - Create a replay buffer to avoid sample correlation
  - Use an auxiliary estimate for q^* (a target network) to avoid bootstrapping
  - Turn the trajectory data into supervised learning data

1. Build replay buffer

- Given a trajectory

$$\mathcal{T} = \{s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{T-1}, a_{T-1}, r_{T-1}, s_T\}$$

create a set of transitions (replay buffer)

$$\mathcal{T}' = \{(s_t, a_t, r_t, s_{t+1}), t = 0, \ldots, T-1\}$$

At training time, we
select random transitions
from the replay buffer

Goal: minimize
sample correlation

2. Build targets

- At training time, given a sample (s_t, a_t, r_t, s_{t+1}) from the replay buffer, build target

$$y_t = r_t + \max_{a \in \mathcal{A}} \hat{q}(s_{t+1}, a)$$

where $\hat{q}$ is an estimate of q^*

Auxiliary estimate
(target network)

- We thus build a dataset

$$\mathcal{D} = \{(s_{t_k}, a_{t_k}, y_{t_k}), k = 1, \dots, K\}$$

3. Train

- The error associated with sample t_k is now

$$\varepsilon_k = (y_{t_k} - q(s_{t_k}, a_{t_k}; \boldsymbol{w}))^2$$

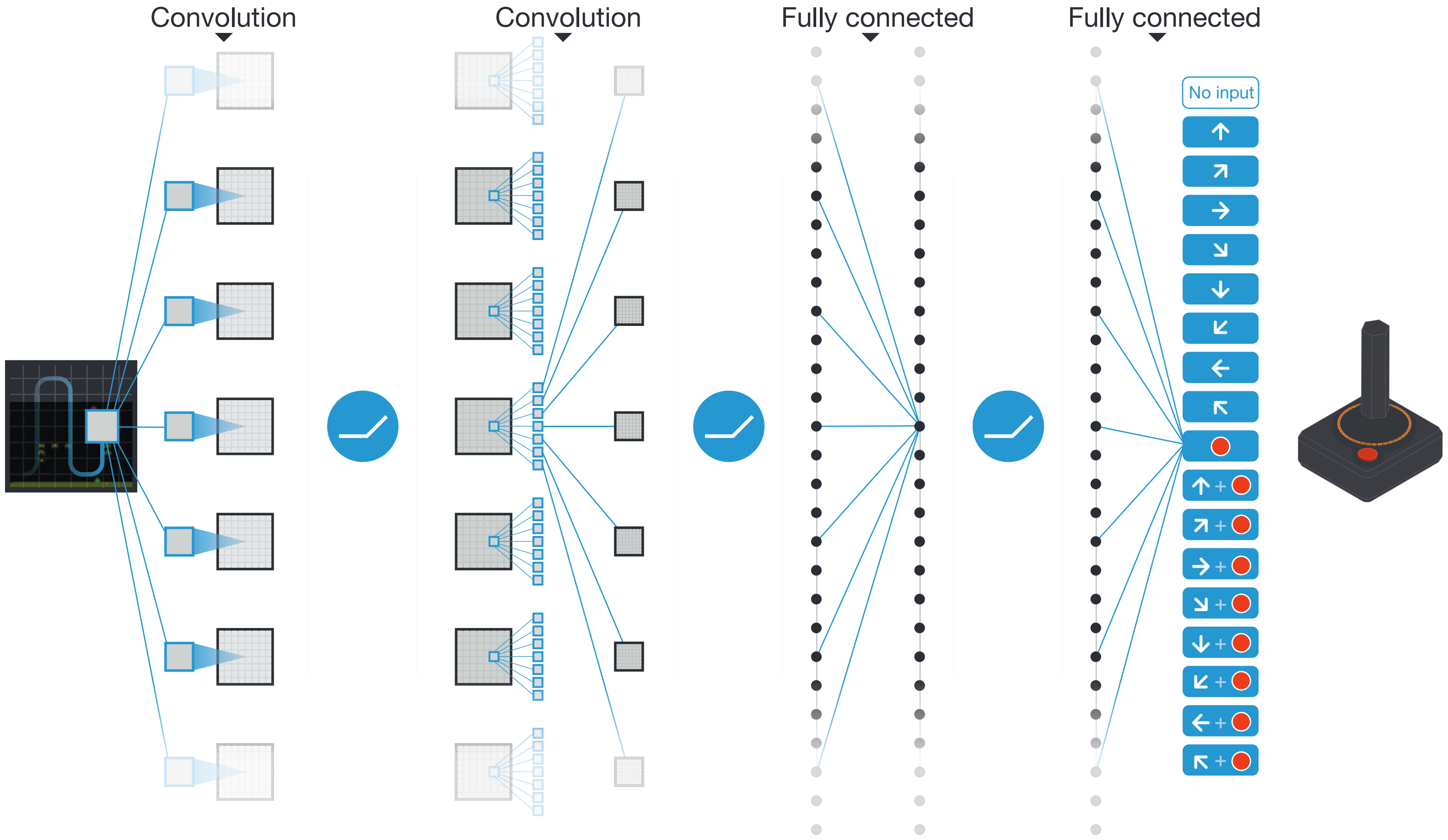
with gradient

$$\begin{aligned}
\nabla_{\boldsymbol{w}} \varepsilon_k &= -2 \nabla_{\boldsymbol{w}} q(s_{t_k}, a_{t_k}; \boldsymbol{w})(y_{t_k} - q(s_{t_k}, a_{t_k}; \boldsymbol{w})) \\
&= \boxed{-2 \nabla_{\boldsymbol{w}} q(s_{t_k}, a_{t_k}; \boldsymbol{w})(r_{t_k} + \gamma \max_{a \in \mathcal{A}} \hat{q}(s_{t+1}, a) - q(s_{t_k}, a_{t_k}; \boldsymbol{w}))}
\end{aligned}$$

Resembles
Q-learning
update

DQN

- The resulting approach is known as a Deep Q-Network (DQN)
- It was the approach used in the ATARI deep RL paper



V. Mnih. “Human-level control through deep reinforcement learning.” *Nature*, 518:529-533, 2015

DQN

- **Some considerations:**
  - The DQN network takes the state as input and has one output per action
  - The target network is a copy of the DQN, i.e.,

“Old” parameters

$$\hat{q}(s,a) = q(s,a;\boldsymbol{w}^{-})$$

  - It is updated every C steps with the weights of the main DQN

Variations: DDQN

- The targets in DQN are computed as

$$y_t = r_t + \max_{a \in \mathcal{A}} q(s_{t+1}, a; \boldsymbol{w}^-)$$

where the target network seeks to avoid bootstrapping

- We can further decouple:
    - ... the computation of the maximizing action; and
    - ... the value of the maximizing action.

Variations: DDQN

- The targets in double DQN (DDQN), the targets are computed as

$$y_t = r_t + q(s_{t+1}, \operatorname*{argmax}_{a \in \mathcal{A}} q(s_{t+1}, a; \boldsymbol{w}); \boldsymbol{w}^{-})$$

Target network is used
to compute the
maximizing value

Original network is used
to compute the
maximizing action

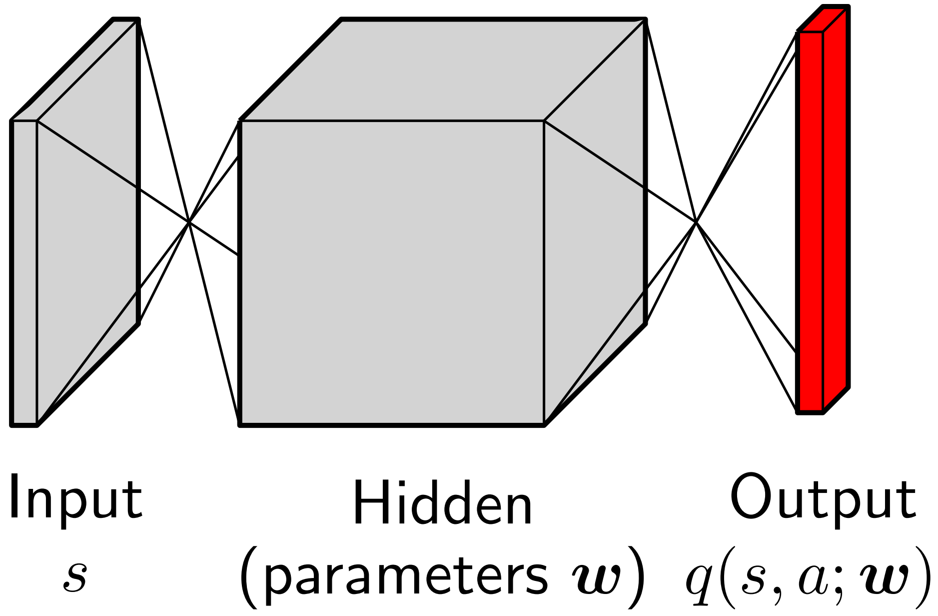
More variations

- Prioritized replay:
  - Transitions are sampled from the replay memory with a probability that increases with the associated error:

$$\varepsilon_k = (y_{t_k} - q(s_{t_k}, a_{t_k}; \boldsymbol{w}))^2$$

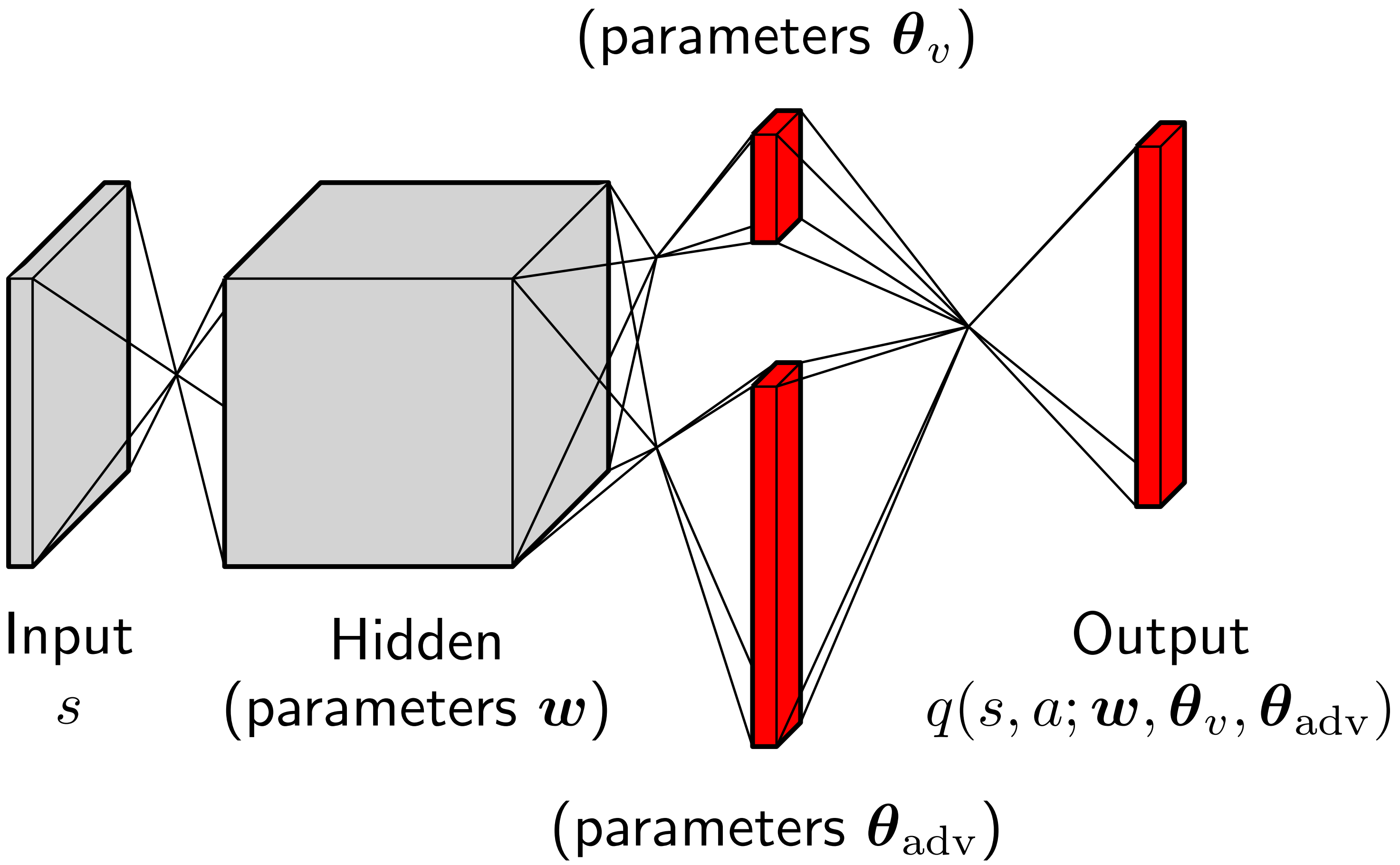
More variations

- Dueling network:
    - Instead of the “standard” DQN architecture



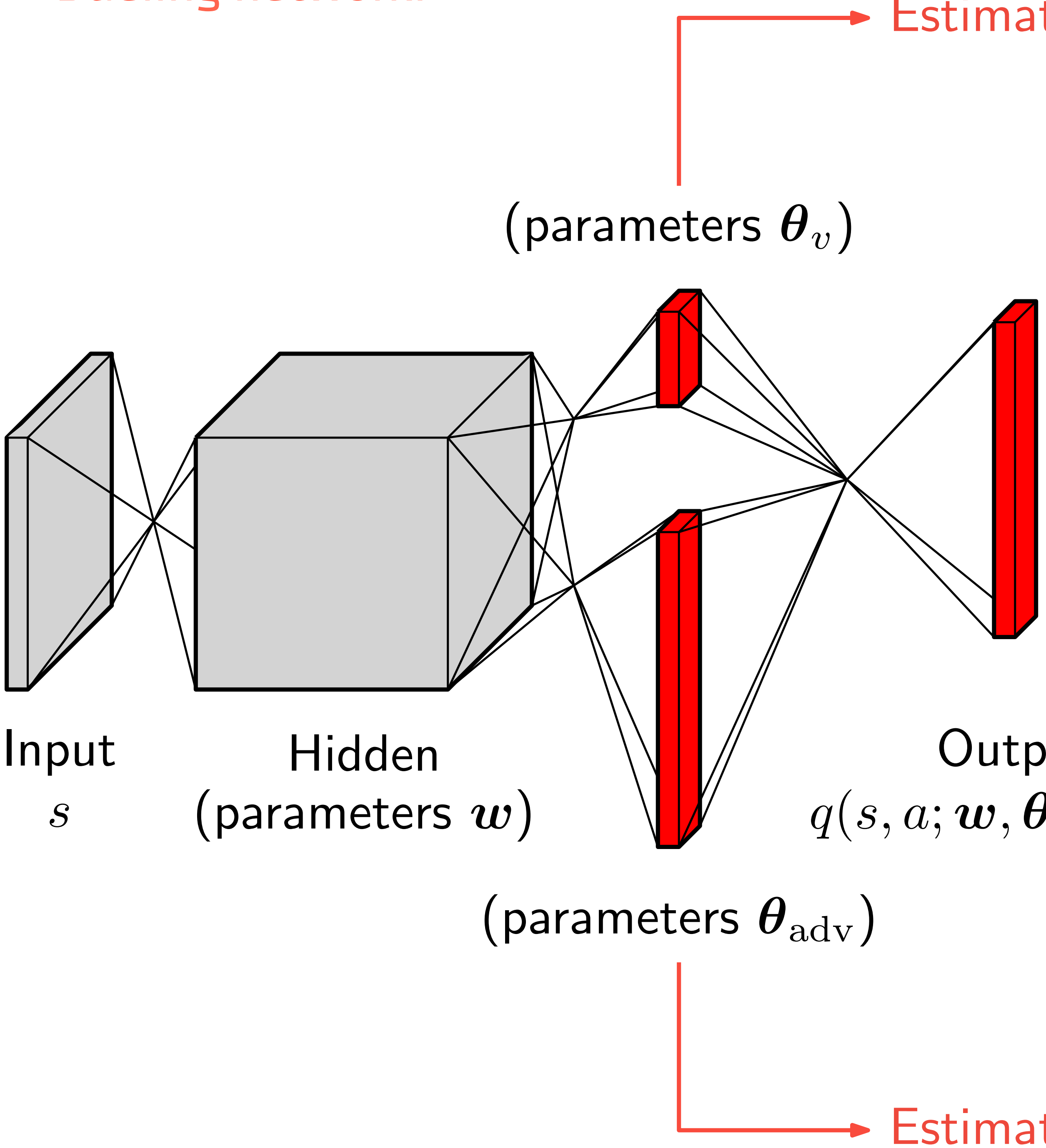
More variations

- Dueling network:
    - Instead of the “standard” DQN architecture, dueling networks propose



More variations

- Dueling network:



Estimate of $v^*(s)$

(parameters $\boldsymbol{\theta}_v$)

Input
s

Hidden
(parameters $\boldsymbol{w}$)

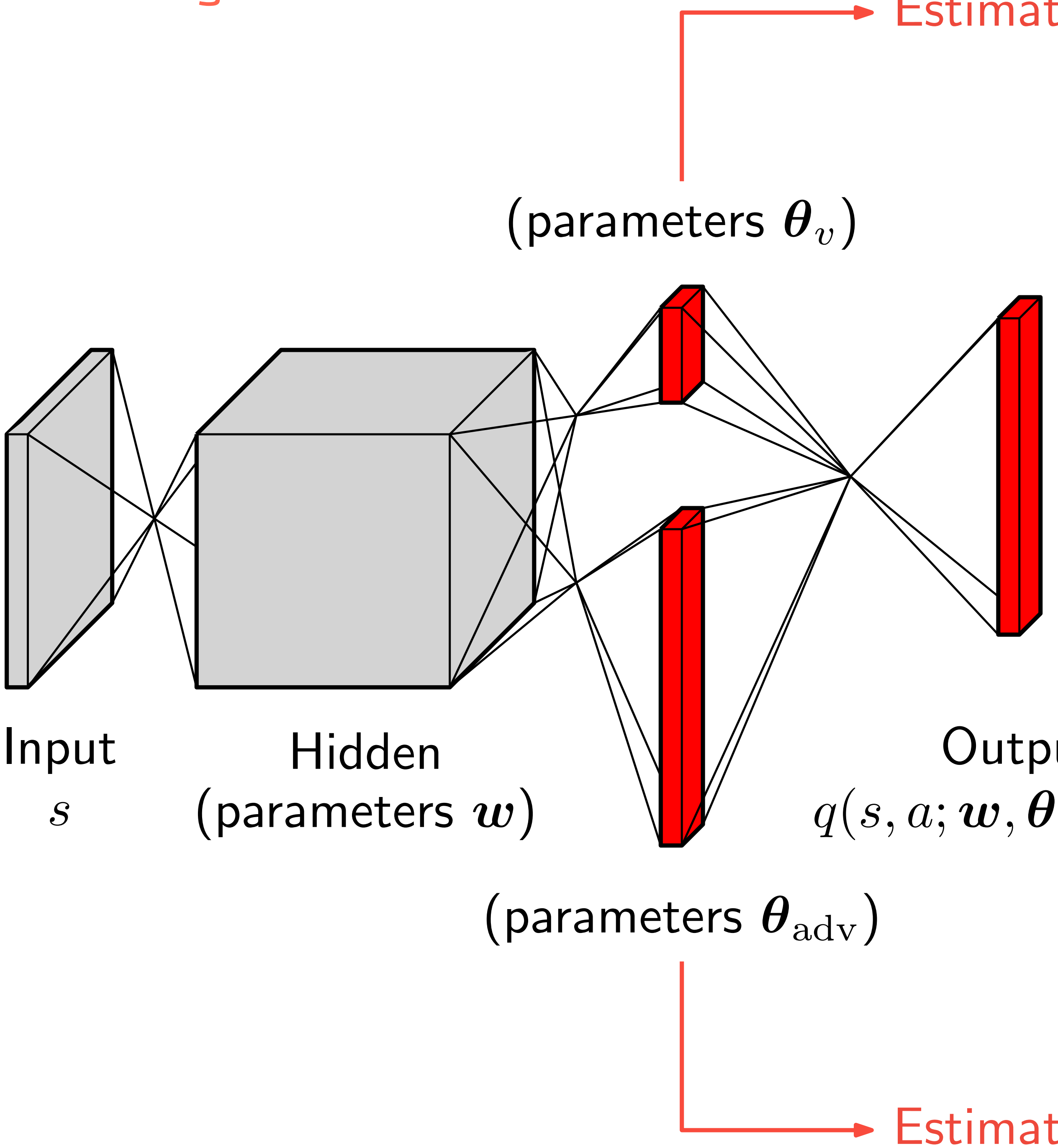
Output
$q(s, a; \boldsymbol{w}, \boldsymbol{\theta}_v, \boldsymbol{\theta}_{\mathrm{adv}}) = v(s; \boldsymbol{w}, \boldsymbol{\theta}_v) + \mathrm{adv}(s, a; \boldsymbol{w}, \boldsymbol{\theta}_{\mathrm{adv}})$

(parameters $\boldsymbol{\theta}_{\mathrm{adv}}$)

Estimate of $\mathrm{adv}^*(s, a)$

More variations

- Dueling network:



Estimate of $v^*(s)$

(parameters $\boldsymbol{\theta}_v$)

Input

s

Hidden

(parameters $\boldsymbol{w}$)

(parameters $\boldsymbol{\theta}_{\mathrm{adv}}$)

Estimate of $\mathrm{adv}^*(s, a)$

Output

$$q(s, a; \boldsymbol{w}, \boldsymbol{\theta}_v, \boldsymbol{\theta}_{\mathrm{adv}}) = v(s; \boldsymbol{w}, \boldsymbol{\theta}_v) + \mathrm{adv}(s, a; \boldsymbol{w}, \boldsymbol{\theta}_{\mathrm{adv}})$$

$$-b(s)$$

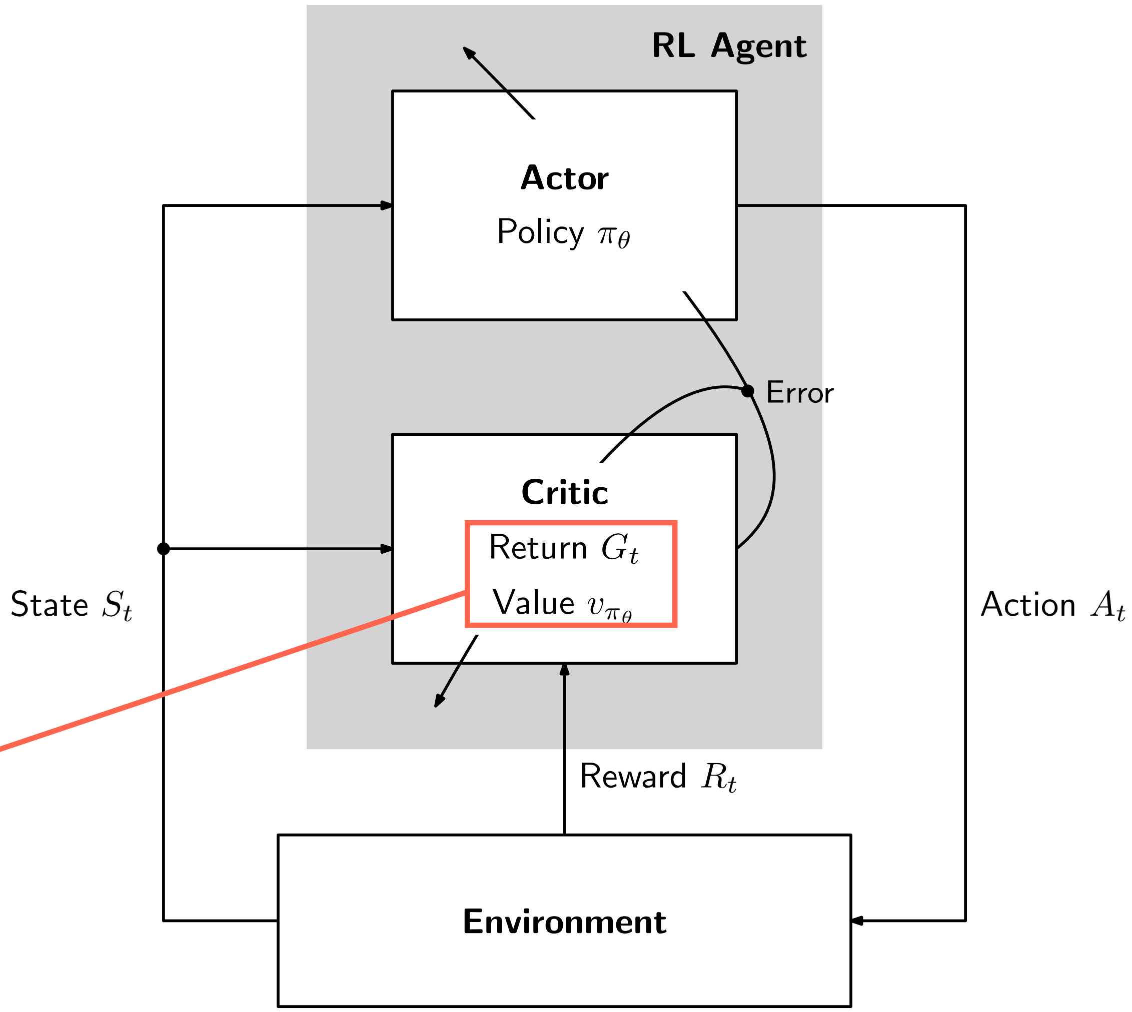
Considerations

- Different variations offer different advantages:
  - DDQN - more stable learning than DQN
  - Prioritized replay - better use of memory (faster learning)
  - Dueling DQN - better performance, particularly in domains where actions only relevant in some states
- Different variations are mostly orthogonal, and can be combined

Policy gradient methods revisited

Actor-critic architecture

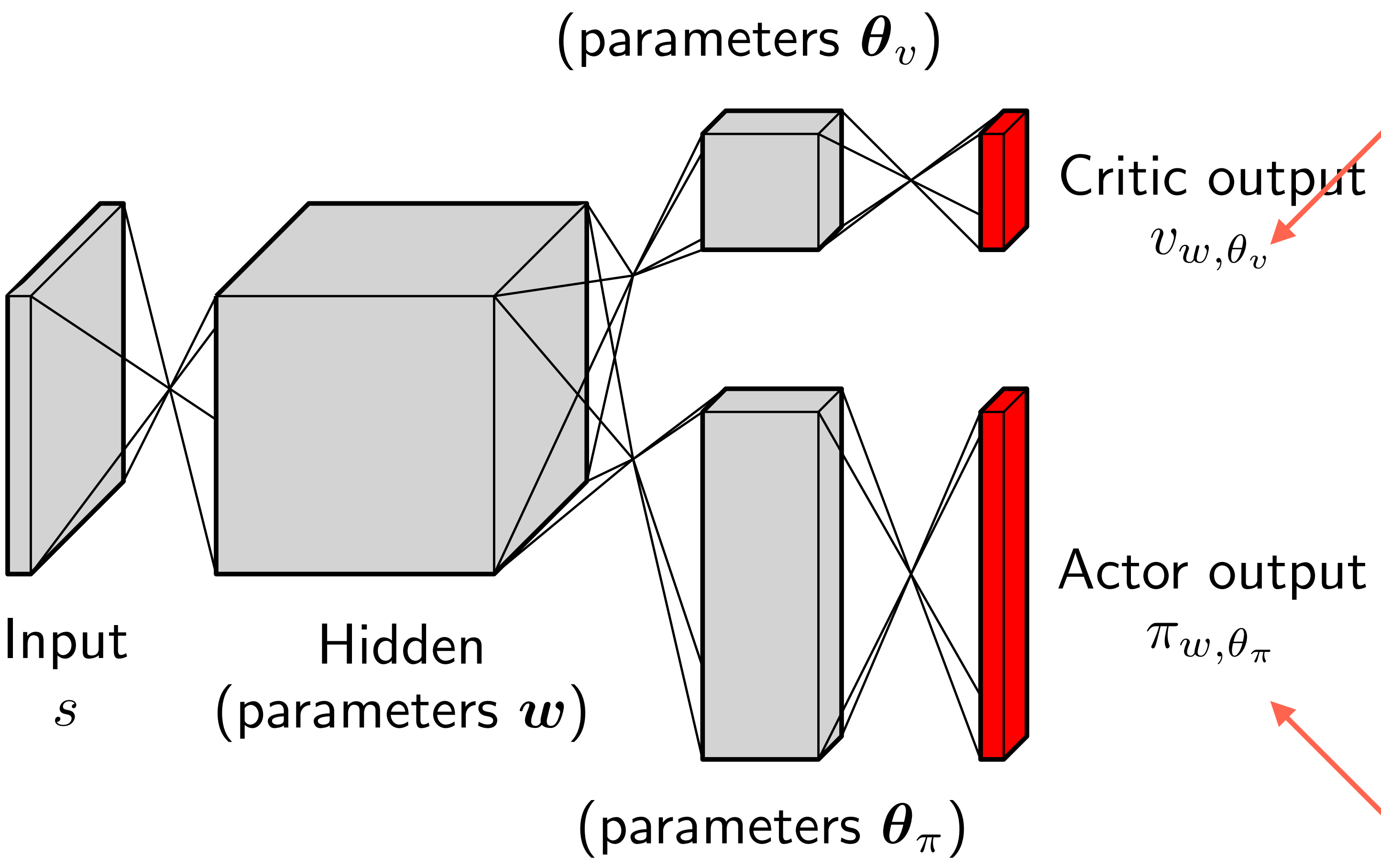
- The AC architecture comprises two components:
  - An actor, responsible for executing the policy π_θ
  - A critic, responsible for evaluating the policy (computing adv_π)



The two components are used to estimate the advantage

Advantage Actor-Critic

$\nabla_{w,\theta_v} v_{w,\theta_v} (G_t - v_{w,\theta_v}(s_t))$



(parameters $\boldsymbol{\theta}_v$)

Critic output
v_{w,θ_v}

Input
s

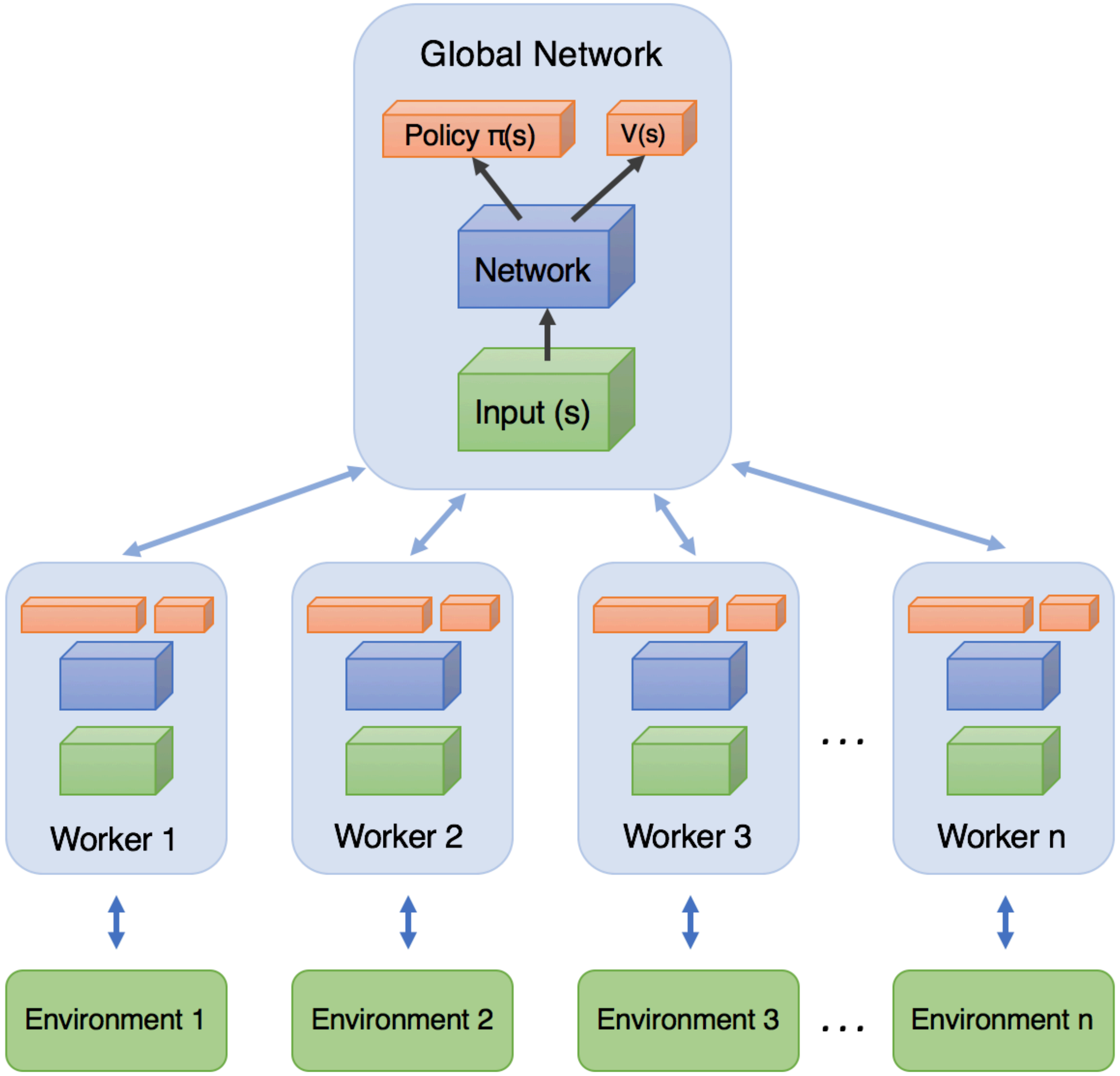
Hidden
(parameters $\boldsymbol{w}$)

Actor output
π_{w,θ_π}

(parameters $\boldsymbol{\theta}_\pi$)

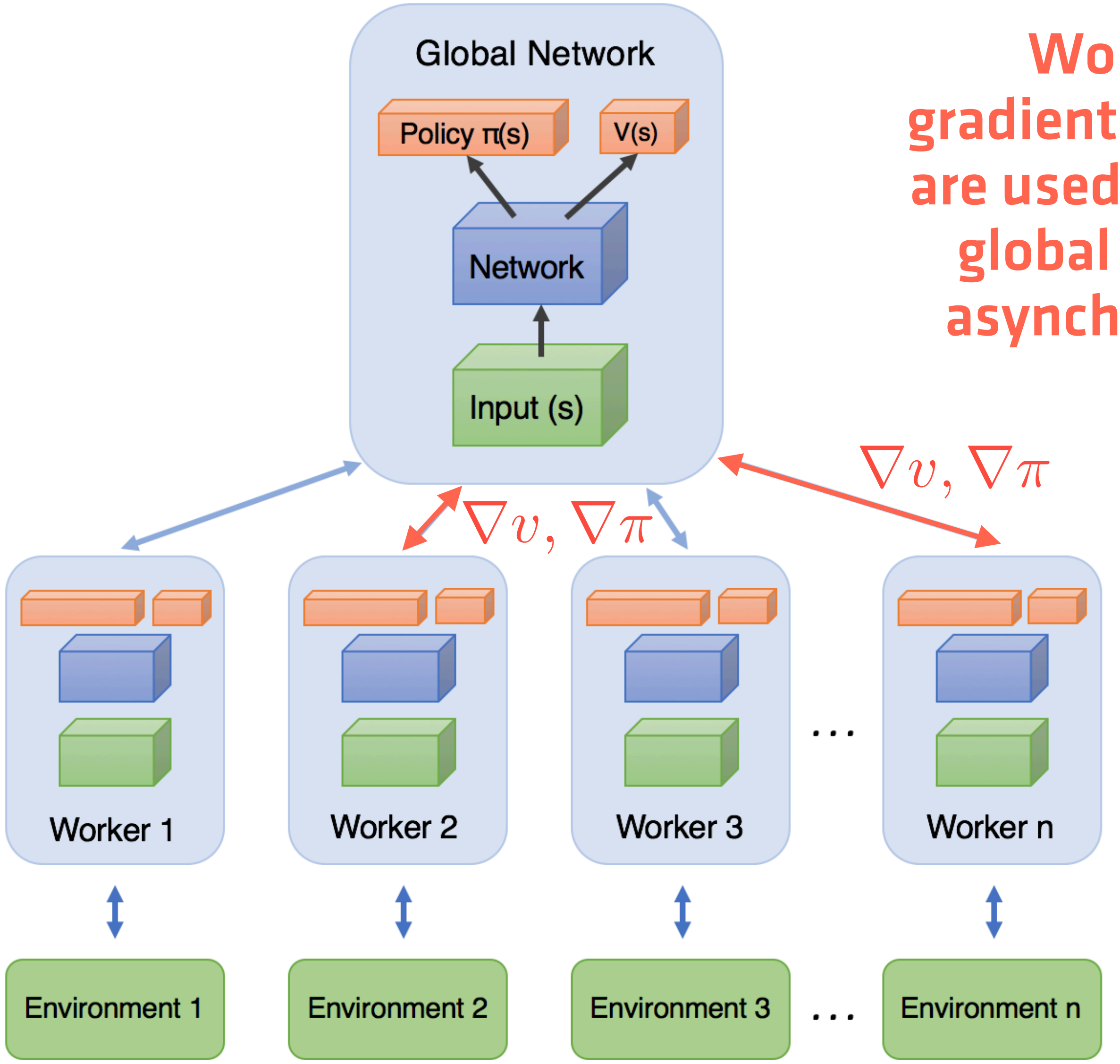
$\gamma^t \nabla_{w,\theta_\pi} \log \pi_{w,\theta_\pi}(a_t \mid s_t)(G_t - v_{w,\theta_v}(s_t))$

Asynchronous Advantage Actor-Critic (A3C)



Workers collect data and compute gradient updates

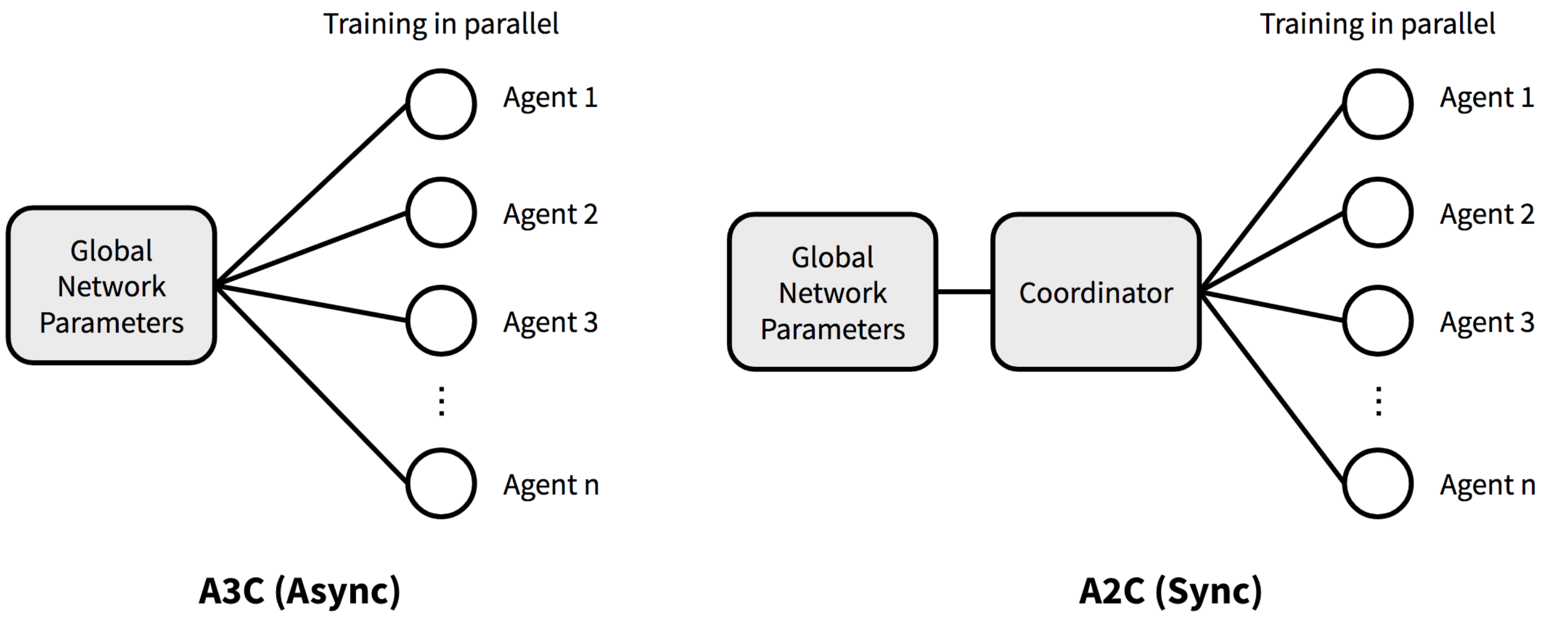
Asynchronous Advantage Actor-Critic (A3C)



Worker's gradient estimates are used to update global network asynchronously

Asynchronous Advantage Actor-Critic (A3C)

- It is not clear that asynchrony brings an advantage
  - Ongoing work to compare A3C with its synchronous version (A2C)
  - A2C includes a coordinator module that ensures that gradient updates are synchronized



Let’s take a step back…

How PG methods work

- Start with a parameterized policy
- Gather some data (trajectories) using that policy
- Use the data to estimate the advantage
- Update policy parameters using the gradient
- Repeat

At this point,
what happens to
the data?

How PG methods work

- Old data is “discarded”
    - Old trajectories may be unlikely under the updated policy
    - Old trajectories provide poor estimate to the advantage under updated policy

↓

Not very
data
efficient

Alternative optimization

- Recall that policy gradient methods arise from the optimization of $J(\pi;\ \mu)$
- Given two policies, π_θ and $\pi_{\theta'}$, it is possible to show that

$$J(\pi_{\theta'}; \mu) = J(\pi_\theta; \mu) + \mathbb{E}_{S \sim \mu_\theta}\left[\sum_{a \in \mathcal{A}} \pi_{\theta'}(a \mid S)\mathrm{adv}_{\pi_\theta}(S, a)\right]$$

Trajectories using π_θ

Advantage weighted by $\pi_{\theta'}$

Alternative optimization

- Recall that policy gradient methods arise from the optimization of $J(\pi;\ \mu)$

- Given two policies, π_θ and $\pi_{\theta'}$, it is possible to show that

$$J(\pi_{\theta'};\mu) = J(\pi_\theta;\mu) + \boxed{\mathbb{E}_{S\sim\mu_\theta}\left[\sum_{a\in\mathcal{A}} \pi_{\theta'}(a \mid S)\mathrm{adv}_{\pi_\theta}(S,a)\right]}$$

  if π_θ and $\pi_{\theta'}$ are "close"

- We can thus optimize $J(\pi_{\theta'};\mu)$ by maximizing the expectation on the r.h.s.

Trust region policy optimization

- TRPO thus consists of solving the optimization problem

$$\begin{aligned} \max_{\theta} \qquad & \mathbb{E}_{S \sim \mu_{\theta_{\text{old}}}} \left[\sum_{a \in \mathcal{A}} \pi_\theta(a \mid S) \text{adv}_{\pi_{\theta_{\text{old}}}}(S, a) \right] \\ \text{subject to} \qquad & \boxed{\mathbb{E}_{S \sim \mu_{\theta_{\text{old}}}} \left[\text{KL}(\pi_{\theta_{\text{old}}}(\cdot \mid S), \pi_\theta(\cdot \mid S)) \right] < \delta} \quad \text{Trust region} \end{aligned}$$

- Can be solved using, e.g., Lagrange multipliers

- How do we compute the expectation?

Estimating the expectation

- We have that

$$\mathbb{E}_{S\sim\mu_{\theta_{\text{old}}}}\left[\sum_{a\in\mathcal{A}}\pi_\theta(a\mid S)\text{adv}_{\pi_{\theta_{\text{old}}}}(S,a)\right]=\mathbb{E}_{S\sim\mu_{\theta_{\text{old}}},A\sim\pi_{\theta_{\text{old}}}}\left[\boxed{\frac{\pi_\theta(A\mid S)}{\pi_{\theta_{\text{old}}}(A\mid S)}}\text{adv}_{\pi_{\theta_{\text{old}}}}(S,A)\right]$$

Same trajectories used in standard PG algorithms

Importance sampling weight

Estimating the expectation

- We have that

$$\mathbb{E}_{S\sim\mu_{\theta_{\text{old}}}}\left[\sum_{a\in\mathcal{A}}\pi_\theta(a\mid S)\text{adv}_{\pi_{\theta_{\text{old}}}}(S,a)\right]=\mathbb{E}_{S\sim\mu_{\theta_{\text{old}}},A\sim\pi_{\theta_{\text{old}}}}\left[\frac{\pi_\theta(A\mid S)}{\pi_{\theta_{\text{old}}}(A\mid S)}\text{adv}_{\pi_{\theta_{\text{old}}}}(S,A)\right]$$

- Right hand side can be estimated from the trajectories

- Interesting fact:

  - If you differentiate the r.h.s. with respect to θ, you get

$$\mathbb{E}_{S\sim\mu_{\theta_{\text{old}}},A\sim\pi_{\theta_{\text{old}}}}\left[\frac{\nabla\pi_\theta(A\mid S)}{\pi_{\theta_{\text{old}}}(A\mid S)}\text{adv}_{\pi_{\theta_{\text{old}}}}(S,A)\right]_{\theta=\theta_{\text{old}}}=\nabla_\theta J(\theta_{\text{old}};\mu)$$

Relation to PG

- If instead of KL divergence we use an Euclidean constraint, i.e.

$$\begin{aligned} \max_{\theta} \quad & \mathbb{E}_{S\sim\mu_{\theta_{\text{old}}}}\left[\sum_{a\in\mathcal{A}} \pi_\theta(a \mid S)\text{adv}_{\pi_{\theta_{\text{old}}}}(S,a)\right] \\ \text{subject to} \quad & \|\theta - \theta_{\text{old}}\|_2^2 < \delta \end{aligned}$$

we recover standard policy gradient

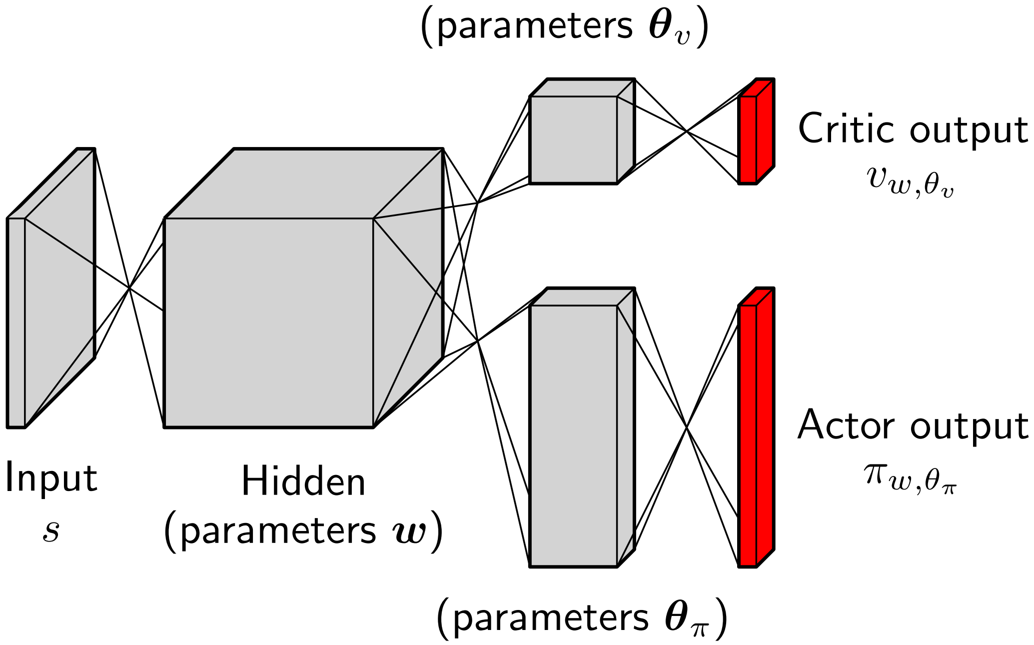
Proximal policy optimization

- Turn the TRPO optimization problem into an unconstrained optimization problem

$$L(\theta) = \mathbb{E}_{S\sim\mu_{\theta_{\text{old}}}, A\sim\pi_{\theta_{\text{old}}}}\left[\frac{\pi_\theta(A \mid S)}{\pi_{\theta_{\text{old}}}(A \mid S)}\text{adv}_{\pi_{\theta_{\text{old}}}}(S, A) - \beta\text{KL}(\pi_{\theta_{\text{old}}}(\cdot \mid S), \pi_\theta(\cdot \mid S))\right]$$

- We can now run SGD on the loss above

- Similar network architecture than standard PG/AC methods



Outline of the lecture

- **Part I: RL Primer**
  - The RL Problem
  - Markov Decision Process - A Model for RL Problems
  - Optimality & Dynamic Programming
  - Monte Carlo Approaches
  - Temporal Difference Learning
  - The Policy Gradient Theorem

Outline of the lecture

- **Part II: Deep RL**
  - From RL to Deep RL
  - DQN
  - Deep advantage actor-critic methods
  - Trust region methods

Conclusion

- Deep learning is an active area of research
- Many recent developments rely on "old" ideas
- Many exploratory works:
  - Algorithmic
  - Architectural
  - Domains

Thank you!

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