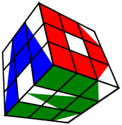


Linear Programming Decoders in NLP:

Integer Programming, Message Passing, Dual Decomposition

André Martins

EMNLP 2014: Tutorials, Doha, Qatar, October 25, 2014

Slides online at `http://tiny.cc/lpdnlp`.

Structured Prediction and NLP

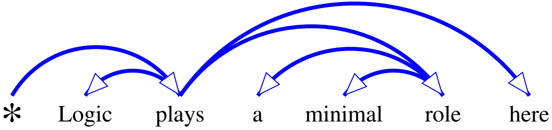
Structured prediction: a machine learning framework for predicting structured, constrained, and interdependent outputs

NLP deals with *structured* and *ambiguous* textual data (Smith, 2011):

- machine translation
- speech recognition
- syntactic parsing
- semantic parsing
- information extraction
- ...

Dependency Parsing

Map **sentences** to their **syntactic structure.**



- A lexicalized syntactic formalism
- Grammar functions represented as lexical relationships (dependencies)

(Eisner, 1996; McDonald et al., 2005; Nivre et al., 2006; Koo et al., 2007)

Multi-Document Summarization

Map a set of related **documents** to a brief **summary.**

CNN

Obama hopes for 'continued progress' in Myanmar

STORY HIGHLIGHTS

- Obama meets with pro-democracy icon Aung San Suu Kyi and Myanmar's president
- He's the first sitting U.S. president to visit Myanmar, also known as Burma
- Obama encourages the country to continue a "remarkable journey"
- He also visits Cambodia to meet the prime minister and attend the East Asia Summit

(CNN) -- Barack Obama met with Nobel Peace Prize winner Aung San Suu Kyi at her home in Myanmar on Monday, praising her "courage and determination" during a historic visit to the once repressive and secretive country.

The first sitting U.S. president to visit Myanmar, Obama urged its leaders, who have embarked on a series of far-reaching political and economic reforms since 2011, not to extinguish the "flickers of progress that we have seen."

Obama said that his visit to the lakeside villa where the pro-democracy icon spent years under house arrest marked a new chapter between the two countries.

"Here, through so many difficult years, is where she has displayed such unbreakable courage and determination," Obama told reporters, standing next to his fellow Nobel peace laureate. "It is here where she showed that human freedom and human dignity cannot be denied."


Myanmar Obama visit

The country, which is also known as Burma, was ruled by military leaders until early 2011 and for decades was politically and economically cut off from the rest of the world.

Suu Kyi acknowledged that Myanmar's opening up would be difficult.

The New York Times

YANGON, Myanmar — President Obama journeyed to this storied tropical outpost of pagodas and jungles on Monday to "extend the hand of friendship" as a land long tormented by repression and poverty begins to throw off military rule and emerge from decades of isolation.

The visit was intended to show support for the reforms put in place by Thein Sein's government since the end of military rule in November 2010.

Activists have warned that the visit may be too hasty - political prisoners remain behind bars and ethnic conflicts in border areas are unresolved.

BBC NEWS

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BBC NEWS

Current State of Affairs

1. Greedy algorithms can deal with rich histories, but they are suboptimal and suffer from error propagation
2. Simple, tractable models permit exact decoding, but they make too stringent factorization assumptions

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1. Greedy algorithms can deal with rich histories, but they are suboptimal and suffer from error propagation
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We'd like *fast* predictors with *global* features and constraints, but how?

Related Recent Tutorials

- Dual Decomposition and Lagrangian Relaxation for Inference in NLP (Rush & Collins ACL 2011)
- Structured Predictions in NLP: Constrained Conditional Models and Integer Linear Programming (Srikumar, Goldwasser & Roth NAACL 2012)
- Variational Inference in Structured NLP Models (Burkett & Klein NAACL 2012)
- Structured Belief Propagation for NLP (Gromley & Eisner ACL 2014)

This Tutorial: Linear Programming Decoders

We'll provide a unified view over these approaches (ILPs, message-passing, dual decomposition)

We'll focus on MAP decoding, but touch briefly on marginal decoding

We'll illustrate with three applications:

1. **Turbo Parsing**
2. **Compressive Summarization**
3. **Joint Coreference Resolution and Quote Attribution**

(Companion software: AD^3 toolkit)

Outline

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1. **Structured Prediction and Factor Graphs**
2. Integer Linear Programming
3. Message-Passing Algorithms
   - Sum-Product
   - Max-Product
4. Dual Decomposition
5. Applications
6. Conclusions

Structured Prediction

- Input set $\mathcal{X}$
- For each $x \in \mathcal{X}$: a **large** set of candidate outputs $\mathcal{Y}(x)$
- A compatibility function $F_{\boldsymbol{w}}(x, y)$ induced by a model $\boldsymbol{w}$
(Linear model: $F_{\boldsymbol{w}}(x, y) = \boldsymbol{w}^\top \boldsymbol{f}(x, y)$)

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- **Training problem:** learn the model $\boldsymbol{w}$ from data $\{\langle x_i, y_i \rangle\}_{i=1}^{M}$
- **Decoding problem (our focus):**

$$\widehat{y} = \arg \max_{y \in \mathcal{Y}(x)} F_{\boldsymbol{w}}(x, y)$$

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- **Key assumption:** $F_{\boldsymbol{w}}$ decomposes into (overlapping) *parts*

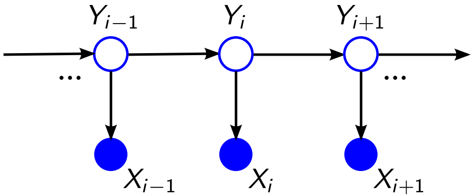
Three Important Questions

- Representation?
- Decoding/Inference?
- Learning the parameters?

Recap: Hidden Markov Models

$F_{\boldsymbol{w}}$ is a log-probability, factoring over emissions and transitions.

$$\mathbb{P}(x, y) = \prod_i \underbrace{\mathbb{P}(x_i | y_i)}_{\text{emissions}} \underbrace{\mathbb{P}(y_i | y_{i-1})}_{\text{transitions}}$$



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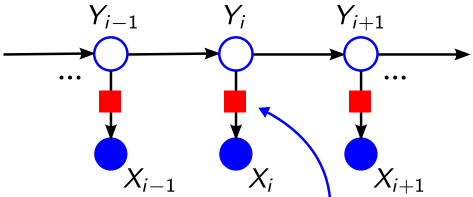

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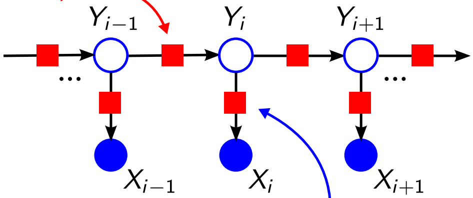

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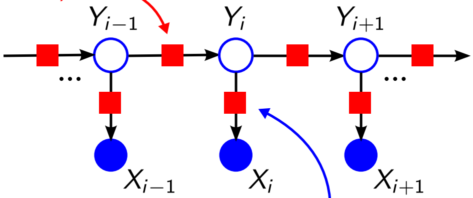

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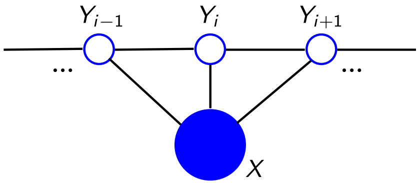
Recap: Hidden Markov Models

- Representation? **Directed sequence model.**
- Decoding/Inference? **Viterbi/forward-backward algorithms.**
- Learning the parameters? **Maximum likelihood (count and normalize).**

Recap: Conditional Random Fields

Same factorization, but globally normalized.

$$\mathbb{P}(y|x) = \frac{1}{Z(\boldsymbol{w}, x)} \exp\left(\sum_i \underbrace{\boldsymbol{w}^\top \boldsymbol{f}_i(x, y_i)}_{\textbf{nodes}} + \sum_i \underbrace{\boldsymbol{w}^\top \boldsymbol{f}_{i,i-1}(x, y_i, y_{i-1})}_{\textbf{edges}} \right)$$



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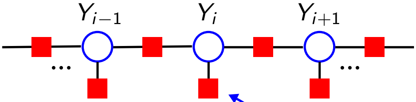

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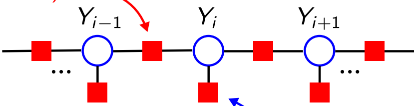

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$$\propto \prod_i \psi_i(y_i) \prod_i \psi_{i,i-1}(y_i, y_{i-1})$$

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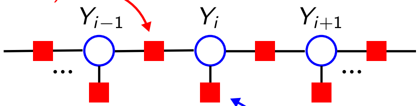


$\psi_i(y_i) := \exp(\mathbf{w}^\top \mathbf{f}(x, y_i))$ (unary potentials)

Recap: Conditional Random Fields

- Representation? **Undirected sequence model.**
- Decoding/Inference? **Viterbi/forward-backward algorithms.**
- Learning the parameters? **Maximum conditional likelihood (convex optimization).**

Graphical Models

HMMs and CRFs are two instances of *graphical models*.

In general, graphical models come in two flavours:

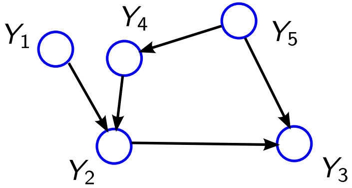
- Directed (Bayesian Networks)
- Undirected (Markov Networks)

Bayesian Networks

Useful to express **causality relations**.

Factors are **conditional probability tables**.

$$\mathbb{P}(y) = \mathbb{P}(y_1)\mathbb{P}(y_2|y_1, y_4)\mathbb{P}(y_3|y_2, y_5)\mathbb{P}(y_4|y_5)\mathbb{P}(y_5)$$



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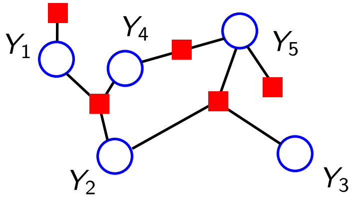


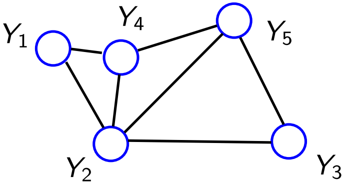
$$\boxed{\mathbb{P}(y) = \prod_i \mathbb{P}(y_i|\text{parents}(y_i))}$$

Markov Networks

Useful to express **correlations between variables**.

Factors correspond to **cliques of the graph**.

$$\mathbb{P}(y) = \frac{1}{Z}\psi_{124}(y_1, y_2, y_4)\psi_{235}(y_2, y_3, y_5)\psi_{245}(y_2, y_4, y_5)$$



$$\boxed{\mathbb{P}(y) \propto \prod_{s \in \text{cliques}(G)} \psi_s(\boldsymbol{y}_s)}$$

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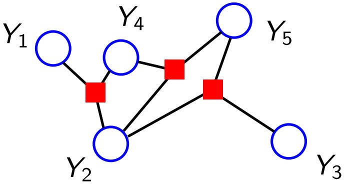

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Conditional Independence

Graphical models are a great tool for modeling conditional independence

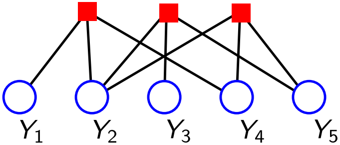
They link properties of the probability distribution with properties of the graph (reachability, D-separation, etc.)

Lots of literature about this: Pearl (1988); Lauritzen (1996); Koller and Friedman (2009)

An Intermediate Representation: Factor Graph

A bipartite graph with **variable nodes** and **factor nodes**

It makes *explicit* the factors of the distribution



$$\mathbb{P}(y) \propto \underbrace{\prod_i \psi_i(y_i)}_{\text{unary potentials}} \times \underbrace{\prod_s \psi_s(\boldsymbol{y}_s)}_{\text{higher-order potentials}}$$

With unary potentials only, all variables would be independent

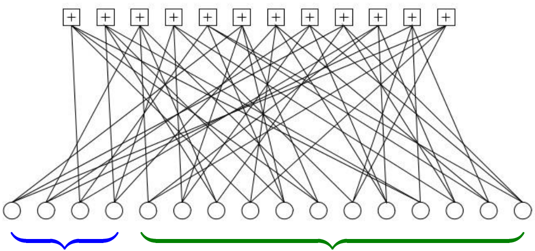
Higher-order potentials can model correlations, impose soft/hard constraints, etc.

Example: Low-Density Parity Check Codes

A message is transmitted through a noisy channel, corrupting some bits

Redundancy can help decoding the message, e.g. via additional parity check bits that can detect/correct errors (error-correcting codes)

High-level idea: increase redundancy to build more accurate decoders



(Adapted from MacKay 2003.)

Inference/Decoding

$$\mathbb{P}_{\psi}(y|x) = \frac{1}{Z(\psi, x)} \times \underbrace{\prod_i \psi_i(y_i)}_{\text{unary potentials}} \times \underbrace{\prod_s \psi_s(\boldsymbol{y}_s)}_{\text{higher-order potentials}}$$

Two decoding problems:

- **MAP decoding:** compute $\widehat{y} = \arg\max_y \mathbb{P}_{\psi}(y|x)$
- **Marginal decoding:** compute every $\mathbb{P}_{\psi}(y_i|x)$ and $\mathbb{P}_{\psi}(\boldsymbol{y}_s|x)$; and evaluate the partition function $Z(\psi, x)$

Sometimes easy, in general intractable...

When is Decoding Easy?

- independent variables (trivial)
- sequence models (Viterbi, forward-backward)
- graphical models without cycles (variable elimination, belief propagation)
- graphical models with low treewidth (junction tree algorithm)

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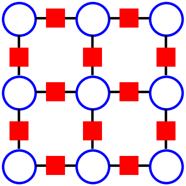
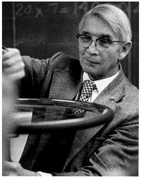
Note: tractability depends not only on the topology, but also on the *potentials*

Example: Ising and Potts Models



Ising/Potts grid



Ernst Ising, 1900–1998

Ren Potts, 1925–2005

All factors are pairwise, variables are binary (Ising) or multi-class (Potts)

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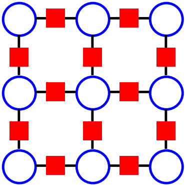

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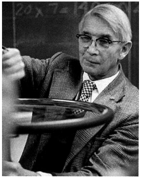

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MAP decoding is tractable for *attractive* Ising models (i.e. Ising models with *supermodular* log-potentials):

$$\log \psi_{ij}(1,1) + \log \psi_{ij}(0,0) \geq \log \psi_{ij}(0,1) + \log \psi_{ij}(1,0)$$

Good approximations for *attractive* Potts models

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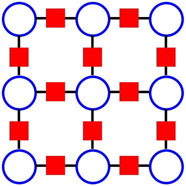

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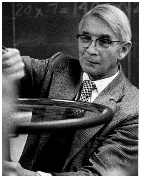

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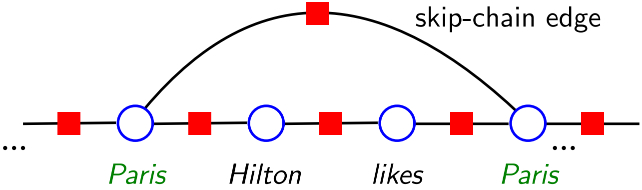
Good approximations for *attractive* Potts models

... but the general case is NP-hard and hard to approximate

Example: Skip-Chain CRFs

Skip-chain CRFs are useful to model long-range dependencies



Skip-chains introduce cycles, making decoding more expensive

We could write this information in the "state" and still decode with dynamic programming, but that would blow up the number of states

Beyond Graphical Models

Some NLP problems (e.g. parsing) require representations beyond graphical models

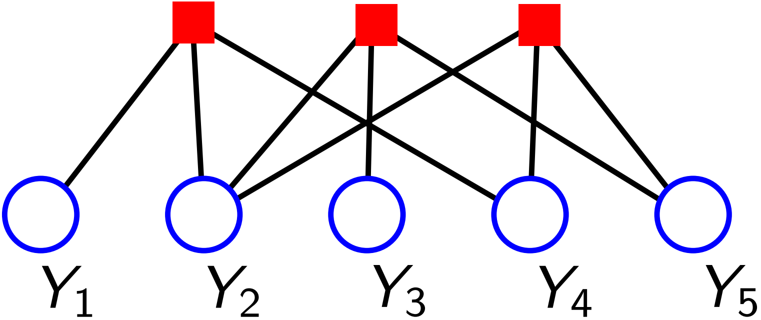
Dynamic programming algorithms (CKY, inside-outside) still work for those representations

Example: case-factor diagrams (McAllester et al., 2008)

Other problems (e.g. matching, spanning trees) can be solved with combinatorial algorithms not related with dynamic programming

All these can still be represented as GMs by "generalizing" the notion of factor

Factors as Machines



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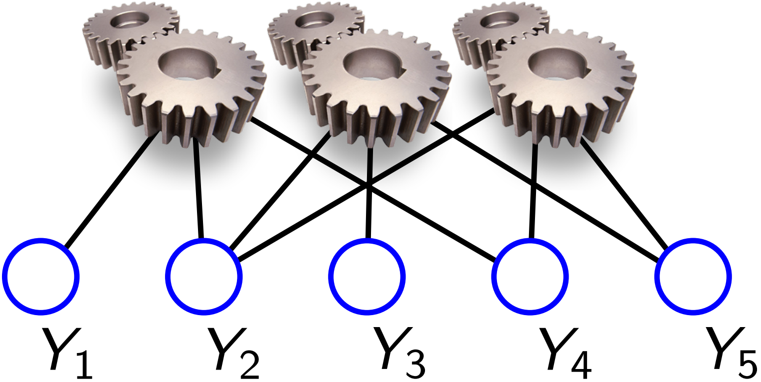
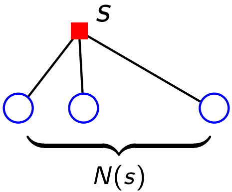


Three Kinds of Factors



Let $N(s)$ denote the set of variables that are *neighbors* of factor s.

(Its cardinality $|N(s)|$ is called the *degree* of s.)

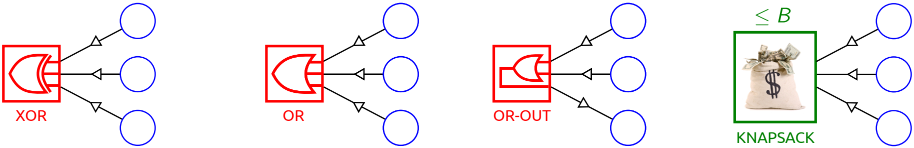
1. **Dense factors:** $\psi_s(\boldsymbol{y}_s)$ has all $O(\exp(|N(s)|))$ degrees of freedom
2. **Structured factors:** $\psi_s(\boldsymbol{y}_s)$ has internal structure
3. **Hard constraint factors:**

$$\psi_s(\boldsymbol{y}_s) := \begin{cases} 1, & \text{if } \boldsymbol{y}_s \in \mathcal{Y}_s \\ 0, & \text{otherwise.} \end{cases}$$

Examples of Structured Factors

- a factor for bipartite matching (Duchi et al., 2007)
- combining a sequential model (POS tagger) with a PCFG (Rush et al., 2010)
- combining CCG parsing and supertagging (Auli and Lopez, 2011)
- dependency parsing with head automata (Smith and Eisner, 2008; Koo et al., 2010)
- handling string-valued variables with factors that are finite state transducers (Dreyer and Eisner, 2009)
- inversion transduction grammar constraint (Burkett and Klein, 2012)

Examples of Hard Constraint Factors



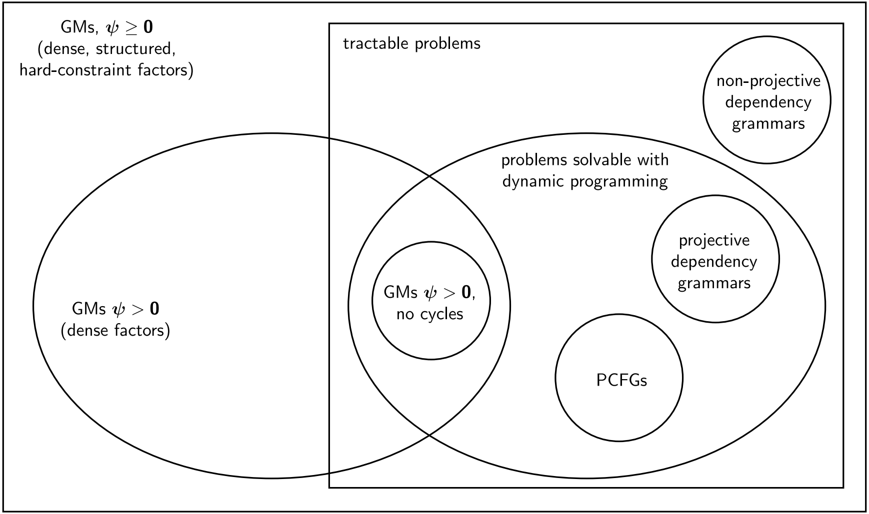
Logic factors: can express arbitrary FOL constraints

- Applications: Markov logic networks (Richardson and Domingos, 2006), constrained conditional models (Roth and Yih, 2004)

Knapsack factors: can express budget constraints

- Applications: summarization, diversity problems,...

(Martins et al., 2011b; Almeida and Martins, 2013; Martins et al., 2014)


GMs, $\psi \geq \mathbf{0}$
(dense, structured,
hard-constraint factors)
tractable problems
non-projective
dependency
grammars
problems solvable with
dynamic programming
projective
dependency
grammars
GMs $\psi > \mathbf{0}$
(dense factors)
GMs $\psi > \mathbf{0}$,
no cycles
PCFGs

Approximate Decoding

What to do when exact decoding is intractable?

- Sampling methods (MCMC, etc.)
- Mean field algorithms
- LP relaxations
- Message-passing
- Dual decomposition

We'll highlight connections between several of these methods.

Approximate Decoding

What to do when exact decoding is intractable?

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We'll highlight connections between several of these methods.

Global/Local Decoding

"Local" denotes independent problems within the scope of each factor

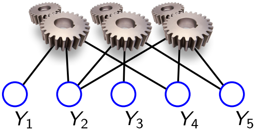
"Global" involves a global assignment of variables, consistent across factors

Key idea: "glue" the local evidence at the factors to obtain a global assignment

Our assumption: local decoding is easy, for every factor

We want to build a good (approximate) *global* decoder by invoking the *local* decoders.

What Kind of Local Decoding Do We Need?

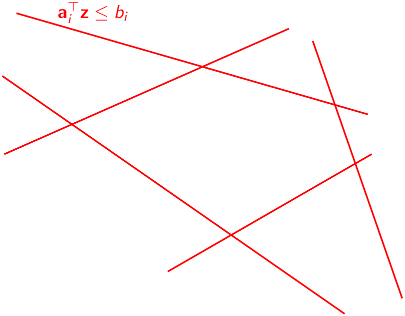


Algorithm	**Local Operation**
Sum-Prod. BP (Pearl, 1988)	marginals
TRBP (Wainwright et al., 2005)	marginals
Norm-Product BP (Hazan and Shashua, 2010)	marginals
Max-Prod. BP (Pearl, 1988)	max-marginals
TRW-S (Kolmogorov, 2006)	max-marginals
MPLP (Globerson and Jaakkola, 2008)	max-marginals
PSDD (Komodakis et al., 2007)	MAP
Accelerated DD (Jojic et al., 2010)	marginals
AD^3 (Martins et al., 2011a)	QP/MAP

Outline

Linear Programming (Kantorovich, 1940; Dantzig, 1947)

$$\max_{\boldsymbol{z}} \quad \boldsymbol{s}^\top \boldsymbol{z} \qquad \textbf{Linear objective}$$
$$\text{s.t.} \quad \boldsymbol{a}_i^\top \boldsymbol{z} \leq b_i, \; i = 1, \ldots, N. \qquad \textbf{Linear constraints}$$



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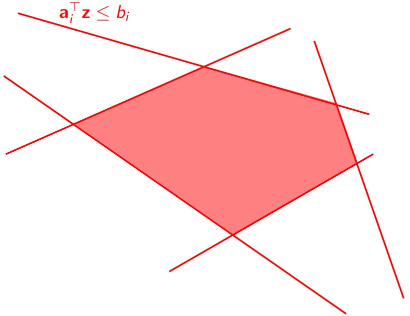

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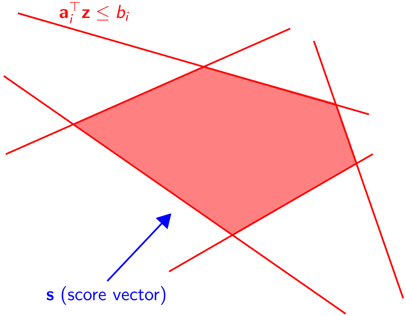
Linear objective

Linear constraints



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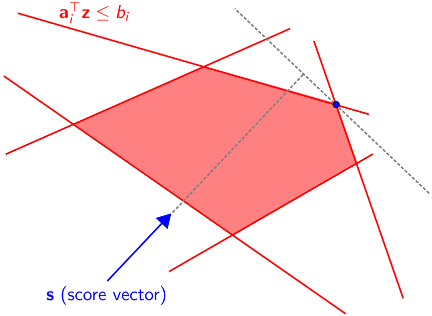

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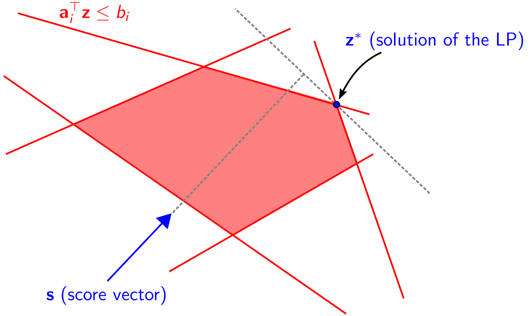
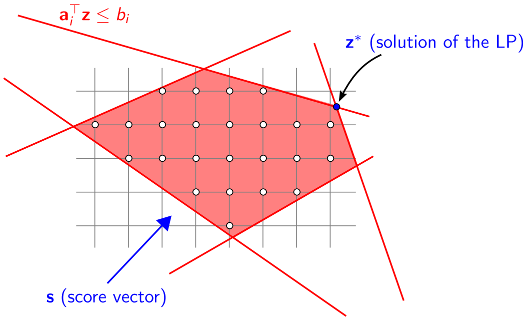


Linear Programming (Kantorovich, 1940; Dantzig, 1947)

- If feasible and bounded, the solution is always attained at a vertex
- Can be solved in **polynomial time** (Khachiyan, 1980)
- Lots of off-the-shelf solvers (CPLEX, Gurobi, GLPK, LP_Solve, etc.)

Integer Linear Programming

$$\begin{array}{lll} \max_{\boldsymbol{z}} & \boldsymbol{s}^\top \boldsymbol{z} & \textbf{Linear objective} \\ \text{s.t.} & \boldsymbol{a}_i{}^\top \boldsymbol{z} \le b_i, \; i = 1, \ldots, N, & \textbf{Linear constraints} \\ & \boldsymbol{z} \text{ integer.} & \end{array}$$



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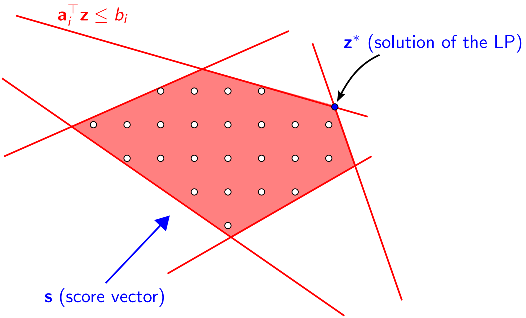

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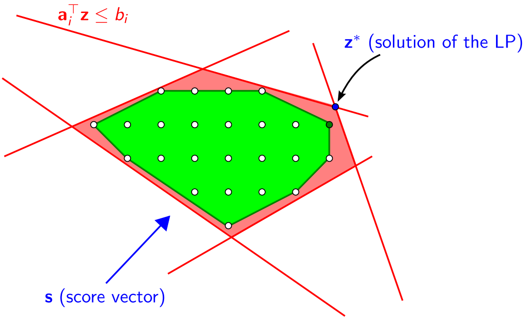

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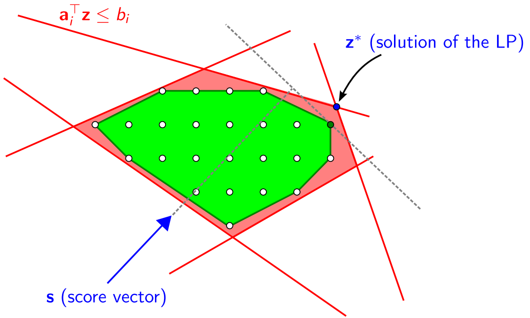

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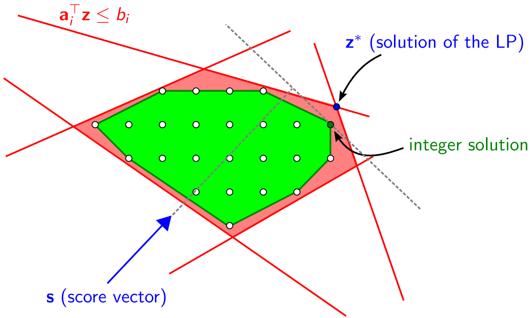


Integer Linear Programming

In general, NP-hard (Karp, 1972)

Existing solvers are effective for small instances, but don't scale

LP relaxation: drops the integer constraints

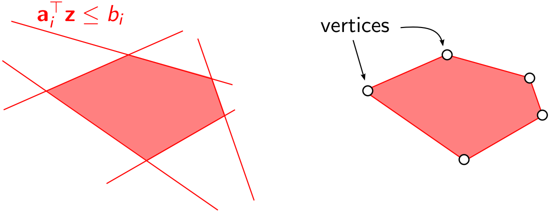
- Gives an upper bound of the solution of the ILP
- A common first step in exact algorithms (branch-and-bound, cutting plane, branch-and-cut)

Here's a very simple approximate algorithm:

1. Solve the LP relaxation
2. If the solution is integer, then it *is* the solution of the ILP
3. Otherwise, apply a rounding heuristic (problem-dependent)

Two Representations of Polytopes

Intersection of half-spaces (**H-representation**) or convex hull of a set of vertices (**V-representation**)

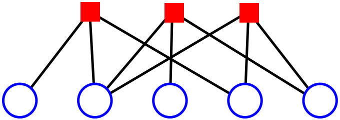


To call a solver, we need to specify a concise H-representation

However, it may be difficult or impossible to obtain one if all we have is a V-representation

We next show how this relates to MAP decoding...

Structured Outputs as Bit-Vectors



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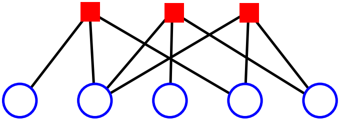
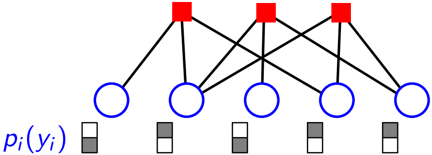


- One indicator $p_i(y_i)$ per each variable state

Structured Outputs as Bit-Vectors



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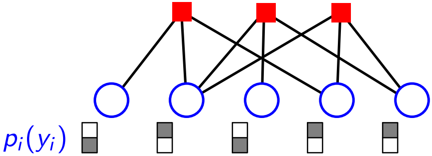

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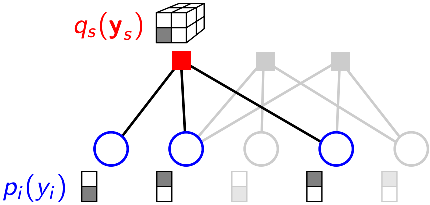

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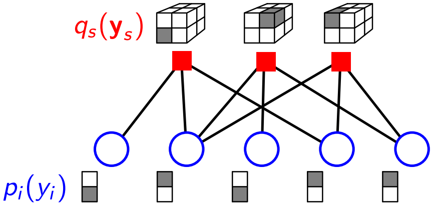

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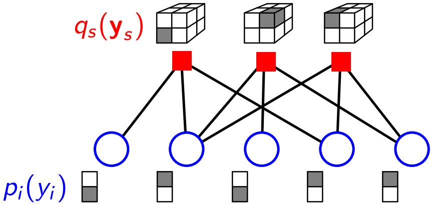

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- Overall: each global output $y \in \mathcal{Y}(x)$ is mapped to a bit-vector

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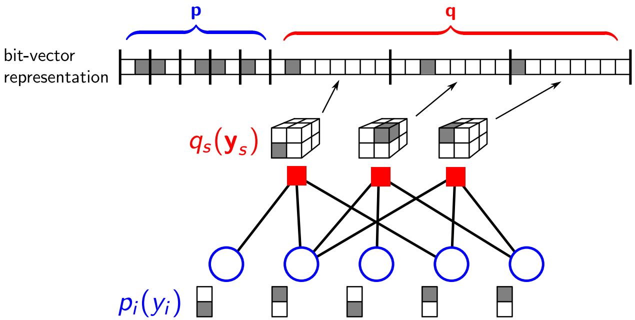

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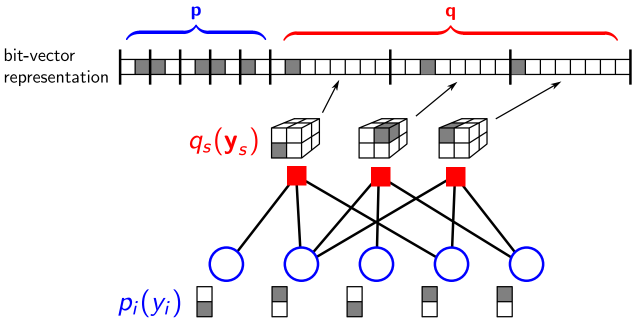

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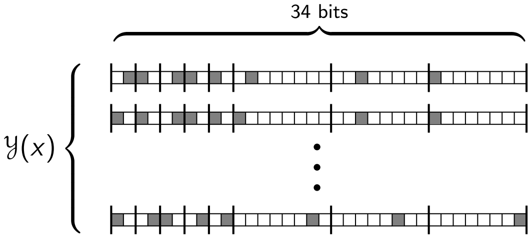
- Overall: each global output $y \in \mathcal{Y}(x)$ is mapped to a bit-vector
- **Note: not all bit vectors are valid (they must be consistent)**

Marginal Polytope (Wainwright and Jordan, 2008)



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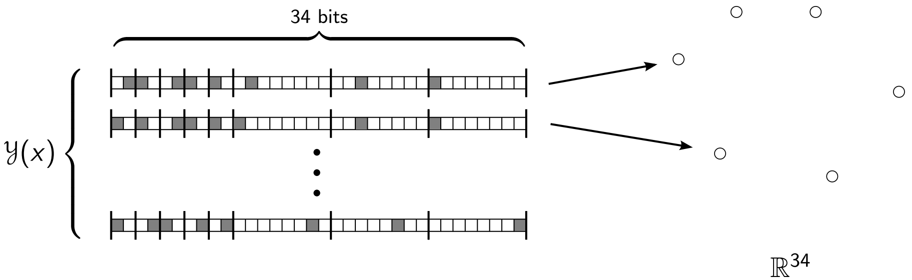

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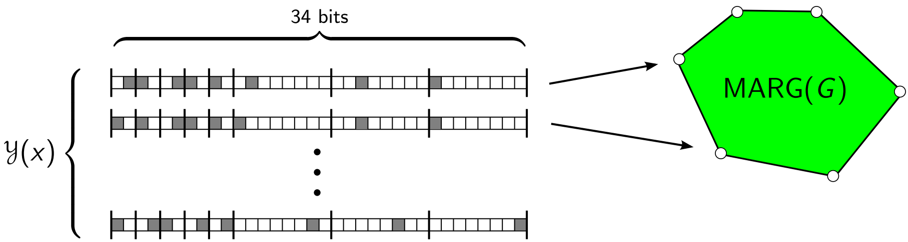

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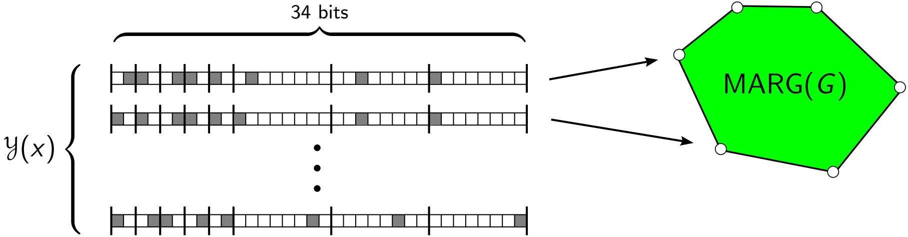

- Vertices of **MARG**(G) correspond to outputs $\mathcal{Y}(x)$
- Points of **MARG**(G) correspond to realizable marginals (more later)

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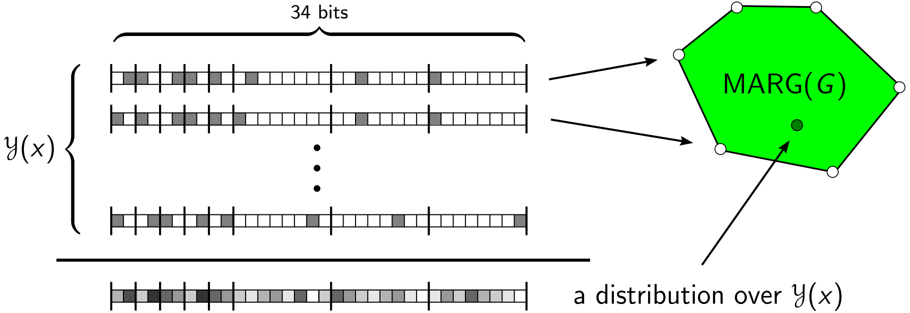

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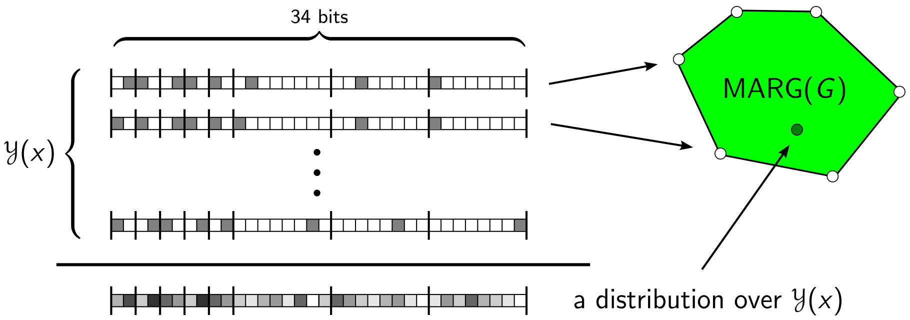


- Vertices of **MARG(G)** correspond to outputs $\mathcal{Y}(x)$
- Points of **MARG(G)** correspond to realizable marginals (more later)
- This is a V-representation, what about an H-representation?

H-Representation With Integer Constraints

In general, there's no concise H-representation for MARG(G)

... but we can represent its vertices if *integer constraints* are permitted:

$$\sum_{\boldsymbol{y}_s} q_s(\boldsymbol{y}_s) = 1, \quad q_s(\boldsymbol{y}_s) \geq \mathbf{0}, \quad \forall \boldsymbol{y}_s \in \mathcal{Y}_s \qquad \textbf{(normalization)}$$

$$p_i(y_i) = \sum_{\boldsymbol{y}_s \sim y_i} q_s(\boldsymbol{y}_s), \quad \forall i \in N(s) \qquad \textbf{(marginalization)}$$

$$\boldsymbol{q} \text{ is integer} \qquad \textbf{(integer constraints)}$$

This will open the door for formulating MAP decoding as an ILP.

MAP Decoding as an ILP

Recall the MAP decoding problem:

$$\begin{aligned}
\widehat{y} &= \arg\max_{y \in \mathcal{Y}(x)} P_{\psi}(y|x) \\
&= \arg\max_{y \in \mathcal{Y}(x)} \cancel{\frac{1}{Z(\psi, x)}} \prod_i \psi_i(y_i) \prod_s \psi_s(\boldsymbol{y}_s) \\
&= \arg\max_{y \in \mathcal{Y}(x)} \sum_i \boldsymbol{\theta}_i(y_i) + \sum_s \boldsymbol{\theta}_s(\boldsymbol{y}_s),
\end{aligned}$$

where $\boldsymbol{\theta}_i(y_i) := \log \psi_i(y_i)$ and $\boldsymbol{\theta}_s(\boldsymbol{y}_s) := \log \psi_s(\boldsymbol{y}_s)$

We can rewrite this as an ILP:

$$\begin{aligned}
&\text{maximize} \quad \sum_i \sum_{y_i} \boldsymbol{\theta}_i(y_i) p_i(y_i) + \sum_s \sum_{\boldsymbol{y}_s} \boldsymbol{\theta}_s(\boldsymbol{y}_s) q_s(\boldsymbol{y}_s) \\
&\text{subject to} \quad (p, q) \in \mathrm{MARG}(G)
\end{aligned}$$

Local Polytope

Obtained by relaxing the integer constraints

Regard $\boldsymbol{p}_i$ and $\boldsymbol{q}_s$ as probability distributions that must be **locally consistent**:

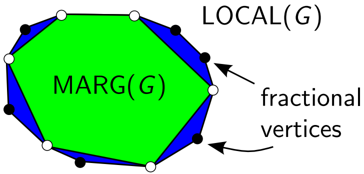
$$\sum_{\boldsymbol{y}_s} q_s(\boldsymbol{y}_s) = 1, \quad q_s(\boldsymbol{y}_s) \geq \mathbf{0}, \quad \forall \boldsymbol{y}_s \in \mathcal{Y}_s \qquad \textbf{(normalization)}$$

$$p_i(y_i) = \sum_{\boldsymbol{y}_s \sim y_i} q_s(\boldsymbol{y}_s), \quad \forall i \in N(s) \qquad \textbf{(marginalization)}$$

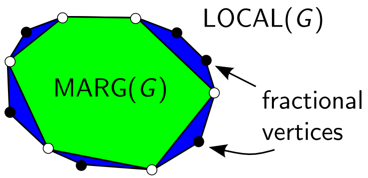
~~$\boldsymbol{q}$ is integer~~ ~~**(integer constraints)**~~

The feasible points are *pseudo-marginals* (not necessarily valid marginals)

Local and Marginal Polytopes



Local and Marginal Polytopes



- LOCAL(G) is an **outer bound** of MARG(G)
- It contains all the integer vertices of MARG(G), plus spurious **fractional vertices**
- **If the graph has no cycles, then LOCAL(G) = MARG(G)**

LP-MAP Decoding

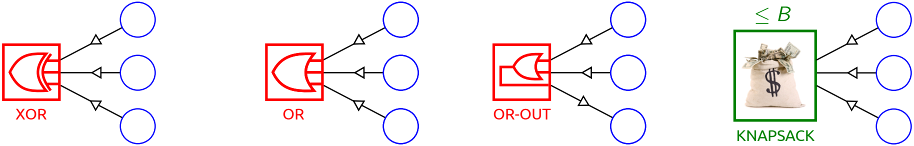
Solves a linear relaxation of MAP decoding, replacing MARG(G) by LOCAL(G):

$$\begin{aligned}\text{maximize} \quad & \sum_i \sum_{y_i} \boldsymbol{\theta}_i(y_i) p_i(y_i) + \sum_s \sum_{\boldsymbol{y}_s} \boldsymbol{\theta}_s(\boldsymbol{y}_s) q_s(\boldsymbol{y}_s) \\ \text{subject to} \quad & (p, q) \in \mathrm{LOCAL}(G)\end{aligned}$$

If the solution is integer, we solved the problem exactly; otherwise, apply a rounding heuristic

Runtime is polynomial, but the procedure is only approximate.

What About Hard Constraint Factors?



Logic and knapsack/budget constraints can also be expressed *linearly*

Logic/Budget Constraints

Assume $z_1, z_2, \ldots \in \{0, 1\}$ (binary variables)

Condition	Statement	Constraint
Implication	if z_1 then z_2	$z_1 \leq z_2$
Negation	z_1 iff $\neg z_2$	$z_1 = 1 - z_2$
OR	z_1 or z_2 or z_3	$z_1 + z_2 + z_3 \geq 1$
XOR	z_1 xor z_2 xor z_3	$z_1 + z_2 + z_3 = 1$
OR-OUT	$z_{12} = z_1 \vee z_2$	$z_{12} \geq z_1, \; z_{12} \geq z_2,$ $z_{12} \leq z_1 + z_2$
AND-OUT	$z_{12} = z_1 \wedge z_2$	$z_{12} \leq z_1, \; z_{12} \leq z_2,$ $z_{12} \geq z_1 + z_2 - 1$
Budget	at most B active units	$\sum_i z_i \leq B$
Knapsack	at most B total weight	$\sum_i w_i z_i \leq B$

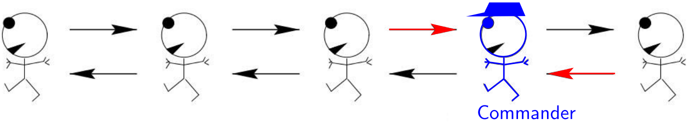
More complex expressions via composition and De Morgan's laws

Summing Up ILPs

- MAP decoding can be expressed as an Integer Linear Program (ILP)
- Logic constraints can be incorporated easily
- Structured factors are harder (they need to be disassembled)
- The ILP can be relaxed for approximate decoding (LP-MAP)
- Geometrically: an outer bound of the *marginal polytope*
- The relaxation is tight if the graph G does not have cycles
- **Disadvantage: an off-the-shelf LP solver won't exploit the *modularity* of the problem**
- **Algorithms that exploit the structure of the LP will be the topic of the remaining sections**

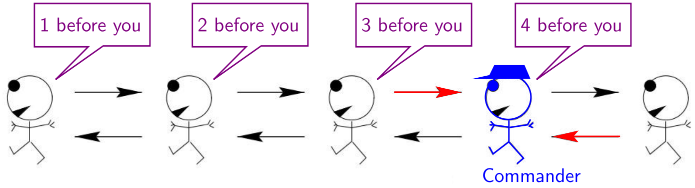
Outline

Motivating Example: Counting Soldiers



(Adapted from MacKay 2003 and Gormley & Eisner ACL'14 tutorial.)

Motivating Example: Counting Soldiers



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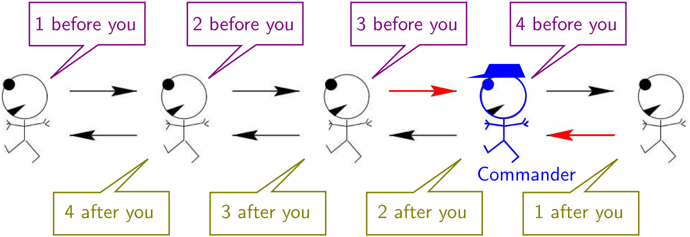

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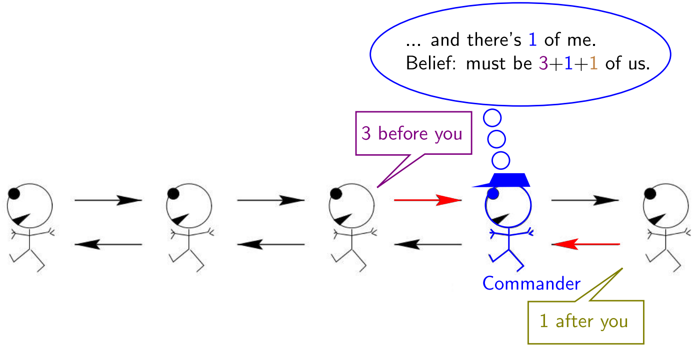
Motivating Example: Counting Soldiers



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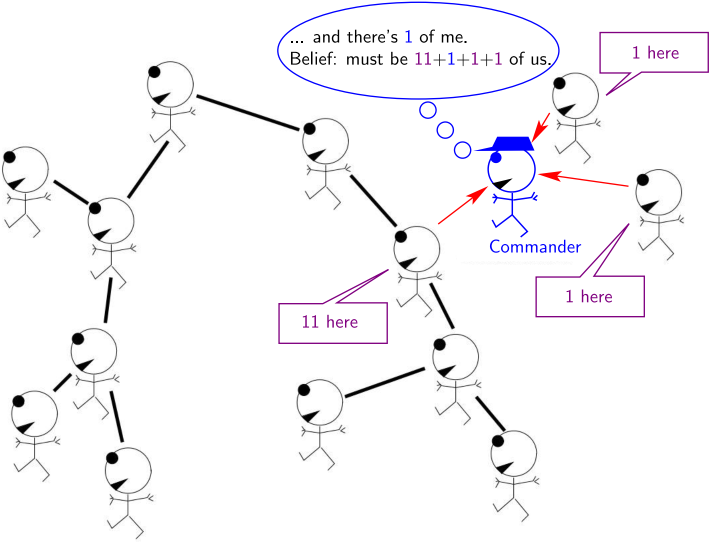

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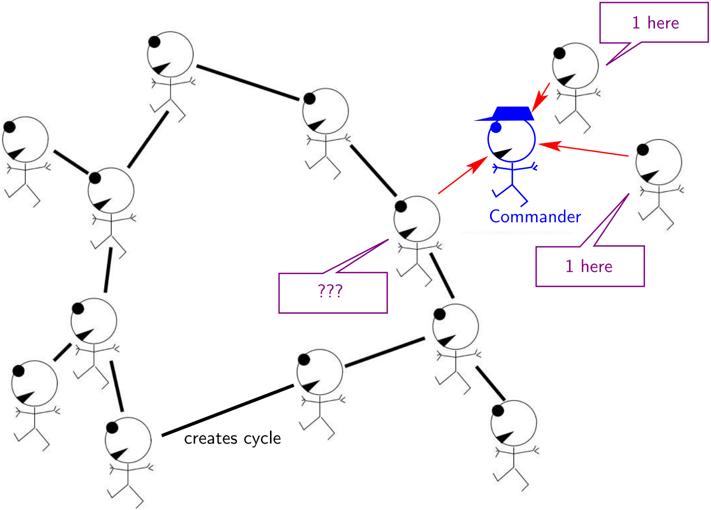

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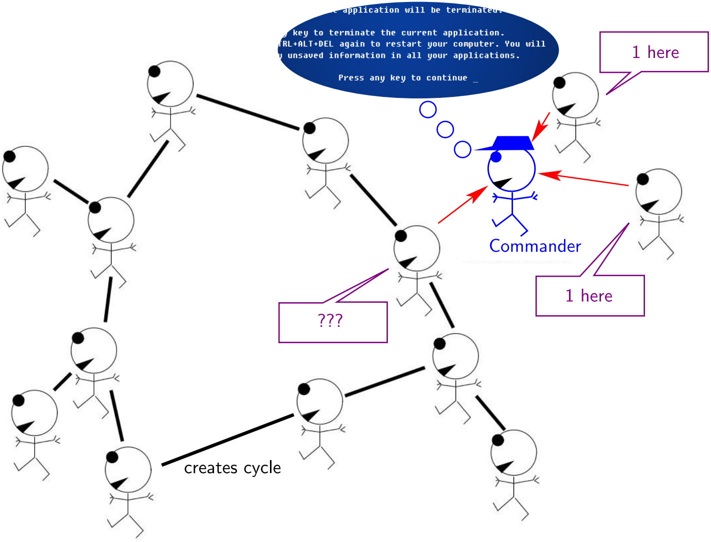

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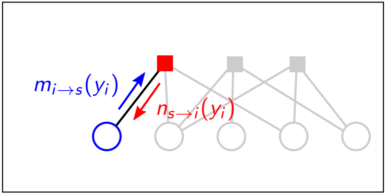
Outline

Sum-Product Belief Propagation

Recall that $\mathbb{P}_{\psi}(y|x) := \frac{1}{Z(\psi, x)} \times \prod_i \psi_i(y_i) \times \prod_s \psi_s(\boldsymbol{y}_s)$

Alternate between computing two kinds of messages:

- **Variable-to-factor:** $m_{i \to s}(y_i) = \psi_i(y_i) \prod_{s' \in N(i) \setminus \{s\}} n_{s' \to i}(y_i)$
- **Factor-to-variable:** $n_{s \to i}(y_i) = \sum_{\boldsymbol{y}_s \sim y_i} \psi_s(\boldsymbol{y}_s) \prod_{j \in N(s) \setminus \{i\}} m_{j \to s}(y_j)$



Beliefs

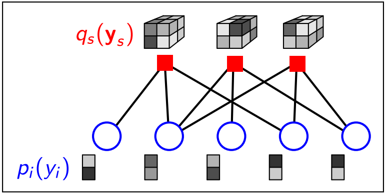
Given the messages, we compute local *beliefs*:

- **Variable beliefs:**

$$p_i(y_i) \propto \psi_i(y_i) \prod_{s \in N(i)} n_{s \to i}(y_i)$$

- **Factor beliefs:**

$$q_s(\boldsymbol{y}_s) \propto \psi_s(\boldsymbol{y}_s) \prod_{i \in N(s)} m_{i \to s}(y_i)$$



If the graph has no cycles, these beliefs converge to the true marginals

$$p_i(y_i) \to \mathbb{P}_\psi(y_i|x), \quad q_s(\boldsymbol{y}_s) \to \mathbb{P}_\psi(\boldsymbol{y}_s|x)$$

Otherwise: loopy BP (later)

Belief Propagation as Calibration

- **Variable-to-factor messages:**

$$m_{i\to s}(y_i) = \psi_i(y_i) \prod_{s' \in N(i) \setminus \{s\}} n_{s'\to i}(y_i) = \frac{p_i(y_i)}{n_{s\to i}(y_i)}$$

- **Factor-to-variable messages:**

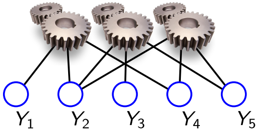
$$n_{s\to i}(y_i) = \sum_{\boldsymbol{y}_s \sim y_i} \psi_s(\boldsymbol{y}_s) \prod_{j \in N(s) \setminus \{i\}} m_{j\to s}(y_j) = \frac{\sum_{\boldsymbol{y}_s \sim y_i} q_s(\boldsymbol{y}_s)}{m_{i\to s}(y_i)}$$

- Calibration equations (attained at convergence):

$$p_i(y_i) = \sum_{\boldsymbol{y}_s \sim y_i} q_s(\boldsymbol{y}_s)$$

Punchline: to run sum-product BP, we only need *local marginals*

What Kind of Local Decoding Do We Need?



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Loopy Belief Propagation

What if the graph has cycles?

Loopy Belief Propagation

What if the graph has cycles?

We'll see that marginal decoding corresponds to optimizing a *free energy objective* over the *marginal polytope*

Sum-product "loopy" BP entails two approximations:

1. Replaces $\text{MARG}(G)$ by $\text{LOCAL}(G)$
2. Optimizes a *Bethe free energy* approximation

Step #1: Dual Parametrization

For any ψ, there are marginals $\boldsymbol{p}$, $\boldsymbol{q}$ in $\mathrm{MARG}(G)$ that parametrize $\mathbb{P}_{\psi}$

E.g. if the graph has no cycles:

$$\begin{aligned}
\mathbb{P}_{\psi}(y|x) &= \frac{1}{Z(\psi, x)} \prod_i \psi_i(y_i) \times \prod_s \psi_s(\boldsymbol{y}_s) \\
&= \prod_i p_i(y_i)^{1-|N(i)|} \times \prod_s q_s(\boldsymbol{y}_s) \qquad \text{(* next slide)} \\
&:= \mathbb{P}_{\boldsymbol{p},\boldsymbol{q}}(y|x)
\end{aligned}$$

Therefore: a distribution can be represented as a point in MARG(G)

$\boldsymbol{\theta} := \log(\psi)$ are called *canonical parameters*, and $(\boldsymbol{p}, \boldsymbol{q})$ *mean parameters*

(*) Derivation of Dual Parametrization

Assume a tree-shaped Bayes net (each variable i has a single parent π_i)

$$
\begin{aligned}
\mathbb{P}(y) &= \mathbb{P}(y_0) \prod_{i \neq 0} \mathbb{P}(y_i | y_{\pi_i}) \\
&= \mathbb{P}(y_0) \prod_{i \neq 0} \frac{\mathbb{P}(y_i, y_{\pi_i})}{\mathbb{P}(y_{\pi_i})} \\
&= \frac{\mathbb{P}(y_0) \prod_s \mathbb{P}(\boldsymbol{y}_s)}{\prod_j \mathbb{P}(y_j)^{|i:j=\pi_i|}} \\
&= \frac{\mathbb{P}(y_0) \prod_s \mathbb{P}(\boldsymbol{y}_s)}{\mathbb{P}(y_0)^{|N(0)|} \prod_{j \neq 0} \mathbb{P}(y_j)^{|N(j)-1|}} \\
&= \frac{\prod_s \mathbb{P}(\boldsymbol{y}_s)}{\prod_j \mathbb{P}(y_j)^{|N(j)|-1}} \\
&= \prod_i p_i(y_i)^{1-|N(i)|} \times \prod_s q_s(\boldsymbol{y}_s).
\end{aligned}
$$

Step #2: Entropy and Log-Partition Function

Entropy of a probability distribution: $H(\mathbb{P}) = -\sum_{y} \mathbb{P}(y) \log \mathbb{P}(y)$

Definition: the Fenchel dual of a convex function $f : \mathbb{R}^D \to \mathbb{R} \cup \{+\infty\}$ is the convex function $f^\star : \mathbb{R}^D \to \mathbb{R} \cup \{+\infty\}$ defined pointwise as
$f^\star(\boldsymbol{v}) := \sup_{\boldsymbol{u}} \left(\boldsymbol{v}^\top \boldsymbol{u} - f(\boldsymbol{u}) \right)$

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Theorem (I): the **log-partition function** $\log Z(\boldsymbol{\theta})$ and the **negative entropy** $-H(\mathbb{P}_{\boldsymbol{p},\boldsymbol{q}})$ are Fenchel dual:

$$\log Z(\boldsymbol{\theta}) = \max_{(\boldsymbol{p},\boldsymbol{q}) \in \mathrm{MARG}(G)} \underbrace{\sum_i {\boldsymbol{\theta}_i}^\top \boldsymbol{p}_i + \sum_s {\boldsymbol{\theta}_s}^\top \boldsymbol{q}_s + H(\mathbb{P}_{\boldsymbol{p},\boldsymbol{q}})}_{\text{(negative) variational free energy}},$$

This underlies the well-known duality between maximum likelihood in log-linear models and maximum entropy.

Step #3: Loopy BP as Variational Inference

Theorem (II): The maximizers $(\boldsymbol{p}^*, \boldsymbol{q}^*)$ are the **true marginals** of $\mathbb{P}_{\boldsymbol{\theta}}$:

$$\boxed{(\boldsymbol{p}^*, \boldsymbol{q}^*) = \arg\max_{(\boldsymbol{p},\boldsymbol{q}) \in \mathrm{MARG}(G)} \sum_i \boldsymbol{\theta}_i{}^\top \boldsymbol{p}_i + \sum_s \boldsymbol{\theta}_s{}^\top \boldsymbol{q}_s + H(\mathbb{P}_{\boldsymbol{p},\boldsymbol{q}})}$$

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Drawback: in general, $\mathrm{MARG}(G)$ and $H(\mathbb{P}_{\boldsymbol{p},\boldsymbol{q}})$ are both intractable

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Drawback: in general, $\mathrm{MARG}(G)$ and $H(\mathbb{P}_{\boldsymbol{p},\boldsymbol{q}})$ are both intractable

Yedidia et al. (2001) showed that loopy BP entails two approximations:

1. Replace $\mathrm{MARG}(G)$ by $\mathrm{LOCAL}(G)$
2. Approximate $H(\mathbb{P}_{\boldsymbol{p},\boldsymbol{q}})$ by the Bethe entropy $H_{\mathrm{Bethe}}(\mathbb{P}_{\boldsymbol{p},\boldsymbol{q}})$

Both are exact when the graph does not have cycles

Bethe Entropy Approximation

Derived by "pretending" the graph has no cycles

We have seen

$$\mathbb{P}_\psi(y|x) \approx \prod_i p_i(y_i)^{1-|N(i)|} \times \prod_s q_s(\boldsymbol{y}_s)$$

Hans Bethe, 1906–2005

From which we can derive

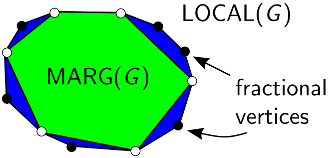
$$\begin{aligned} H(\mathbb{P}_{\boldsymbol{p},\boldsymbol{q}}) &\approx H_{\text{Bethe}}(\mathbb{P}_{\boldsymbol{p},\boldsymbol{q}}) \\ &= \sum_i (1 - |N(i)|) H_i(\boldsymbol{p}_i) + \sum_s H_s(\boldsymbol{q}_s) \end{aligned}$$

A linear combination of local entropies:

$$H_i(\boldsymbol{p}_i) = -\sum_{y_i} p_i(y_i) \log p_i(y_i), \quad H_s(\boldsymbol{q}_s) = -\sum_{\boldsymbol{y}_s} q_s(\boldsymbol{y}_s) \log q_s(\boldsymbol{y}_s)$$

Not concave in general!

Geometric Illustration



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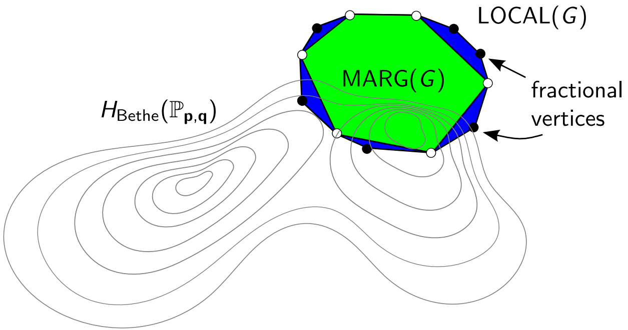

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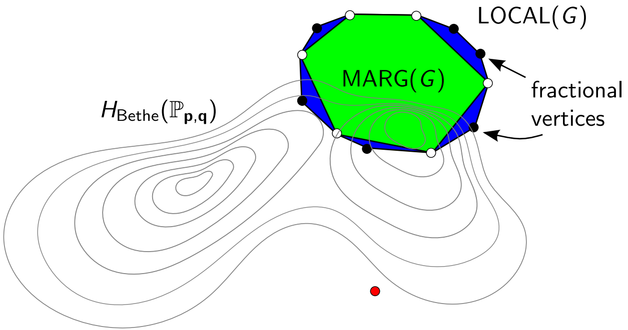

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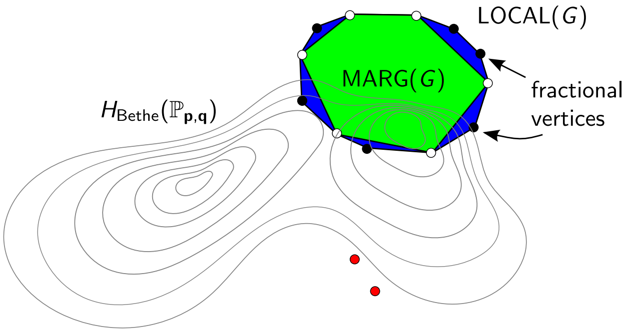

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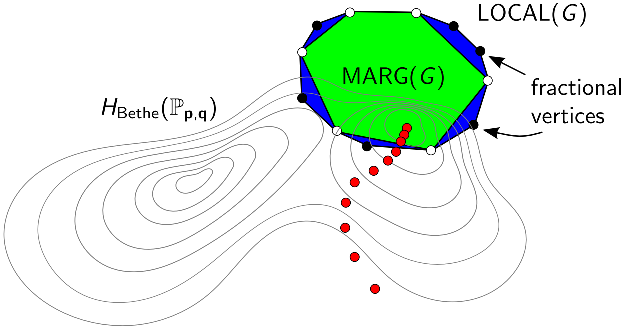

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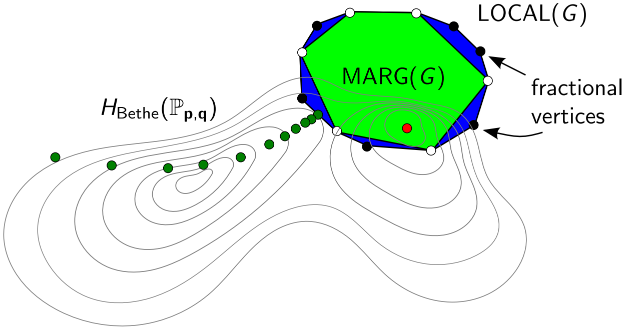

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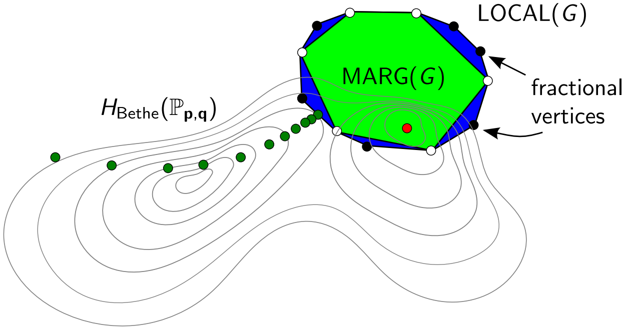


If loopy BP converges, it reaches a stationary point of the approximate variational problem

$H_{\text{Bethe}}(\mathbb{P}_{p,q})$ **is non-concave in general** $\Rightarrow$ **local minima**

Summary of Loopy BP

Advantages:

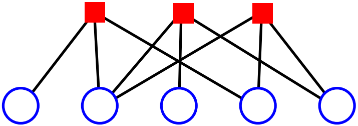
- Simple to implement
- Handles structured and logic factors (only need to compute local marginals)
- Often works well in practice (if cycles are not very influential)
- Often yields a reasonable approximation of $\log Z$ and H

Disadvantages:

- Doesn't give an upper/lower bound of $\log Z$ and H
- Entropy approximation is not concave (local minima)
- May not converge
- The final beliefs may not be realizable marginals

Tree Reweighted BP (Wainwright et al., 2005)

Key idea: cover the graph with a set of trees



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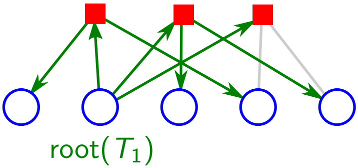

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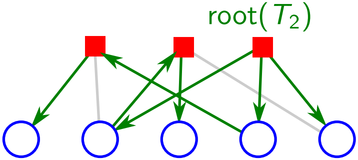

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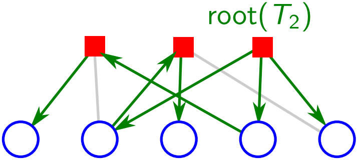

Count the appearance probability $c_{is} > 0$ of each edge

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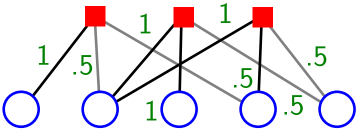

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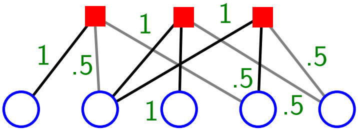


Count the appearance probability $c_{is} > 0$ of each edge

This results in a convex upper bound of $-H$ and $\log Z$:

$$H_{\mathrm{TRBP}}(\mathbb{P}_{\boldsymbol{p},\boldsymbol{q}}) = \sum_i \left(1 - \sum_{s \in N(i)} c_{is}\right) H_i(\boldsymbol{p}_i) + \sum_s H_s(\boldsymbol{q}_s)$$

(Note: if all $c_{is} = 1$ this would revert to the Bethe approximation)

TRBP Messages

- **Variable-to-factor messages:**

$$m_{i\to s}(y_i) = \frac{\psi_i(y_i)\prod_{s'\in N(i)} n_{s'\to i}^{c_{is'}}(y_i)}{n_{s\to i}(y_i)}$$

- **Factor-to-variable messages:**

$$n_{s\to i}(y_i) = \sum_{\boldsymbol{y}_s \sim y_i} \frac{\psi_s(\boldsymbol{y}_s)\prod_{j\in N(s)} m_{j\to s}^{c_{js}}(y_j)}{m_{i\to s}(y_i)}$$

- **Variable beliefs:**

$$p_i(y_i) \propto \psi_i(y_i) \prod_{s\in N(i)} n_{s\to i}^{c_{is}}(y_i)$$

- **Factor beliefs:**

$$q_s(\boldsymbol{y}_s) \propto \psi_s(\boldsymbol{y}_s) \prod_{i\in N(s)} m_{i\to s}^{c_{is}}(y_i)$$

Summary of TRBP

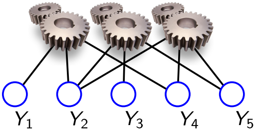
Advantages:

- Still simple to implement
- Entropy approximation is concave (no local minima)
- Gives an upper bound on $-H$ and $\log Z$
- Lots of knobs (the appearance probabilities)

Disadvantages:

- Lots of knobs (the appearance probabilities)
- Typically it's a very loose bound
- May not converge (but in practice always does, with dampening)
- The final beliefs may not be realizable marginals

What Kind of Local Decoding Do We Need?



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AD^3 (Martins et al., 2011a)	QP/MAP

Norm-Product BP (Hazan and Shashua, 2010)

Subsumes loopy BP and TRBP

Relies on a convex approximation to the entropy using *counting numbers* $c_i \geq 0$ and $c_s > 0$ (in its simpler variant)

$$H_{\text{NPBP}}(\mathbb{P}_{\boldsymbol{p},\boldsymbol{q}}) = \sum_i c_i H_i(\boldsymbol{p}_i) + \sum_s c_s H_s(\boldsymbol{q}_s)$$

Messages will become *norms*

Recall the definition of ℓ_p-norm: $\|\boldsymbol{x}\|_p = \left(\sum_i |x_i|^p\right)^{1/p}$

NPBP Messages

- **Variable-to-factor messages:**

$$m_{i\to s}(y_i) = \frac{\left(\psi_i(y_i)\prod_{s'\in N(i)} n_{s'\to i}(y_i)\right)^{c_s/(c_i+\sum_{s'\in N(i)} c'_s)}}{n_{s\to i}(y_i)}$$

- **Factor-to-variable messages:**

$$n_{s\to i}(y_i) = \left(\sum_{\boldsymbol{y}_s\sim y_i}\left(\psi_s(\boldsymbol{y}_s)\prod_{j\in N(s)\setminus\{i\}} m_{j\to s}(y_j)\right)^{1/c_s}\right)^{c_s}$$

- **Variable beliefs:**

$$p_i(y_i) \propto \left(\psi_i(y_i)\prod_{s\in N(i)} n_{s\to i}(y_i)\right)^{1/(c_i+\sum_{s'\in N(i)} c'_s)}$$

- **Factor beliefs:**

$$q_s(\boldsymbol{y}_s) \propto \left(\psi_s(\boldsymbol{y}_s)\prod_{i\in N(s)} m_{i\to s}(y_i)\right)^{c_s}$$

Summary of NPBP

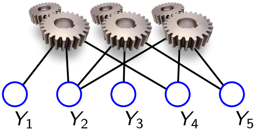
Advantages:

- Still simple to implement
- Entropy approximation is concave (no local minima)
- Always converges (primal-dual block ascent)
- Lots of knobs (the counting numbers)

Disadvantages:

- Lots of knobs (the counting numbers)
- Messages are not computed in parallel (otherwise, may not converge)
- The final beliefs may not be realizable marginals

What Kind of Local Decoding Do We Need?



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Outline

Zero-Limit Temperature

Define Z_ϵ where ϵ is a *temperature parameter*:

$$Z_\epsilon(\psi, x) = \left(\sum_{y \in \mathcal{Y}(x)} \prod_i \psi_i(y_i)^{1/\epsilon} \prod_s \psi_s(y_s)^{1/\epsilon} \right)^\epsilon$$

If $\epsilon = 1$, this becomes the partition function $Z(\psi, x)$

If $\epsilon \to 0$, this becomes the mode of $\mathbb{P}_\psi(y|x)$

Note that $Z_\epsilon(\psi, x) = Z(\psi^{1/\epsilon}, x)^\epsilon$ for any ϵ, i.e., Z_ϵ **can be computed by the same means as the partition function by scaling the potentials**

By choosing a small enough ϵ, any sum-product message-passing algorithm can be used to approximate the MAP

There is a trade-off between precision and numerical stability

Max-Product Belief Propagation

- For MAP decoding instead of marginal decoding
- Only change: factor-to-variable messages (max instead of $\sum$)

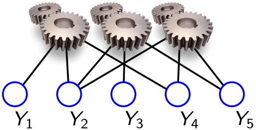
$$n_{s\to i}(y_i) = \max_{\boldsymbol{y}_s \sim y_i} \left(\psi_s(\boldsymbol{y}_s) \prod_{j \in N(s)\setminus\{i\}} m_{j\to s}(y_j) \right) = \frac{\max_{\boldsymbol{y}_s \sim y_i} q_s(\boldsymbol{y}_s)}{m_{i\to s}(y_i)}$$

- If the graph has no cycles, beliefs will converge to **max-marginals**:

$$p_i(y_i) \to \max_{y \sim y_i} \mathbb{P}_\psi(y|x), \quad q_s(\boldsymbol{y}_s) \to \max_{y \sim \boldsymbol{y}_s} \mathbb{P}_\psi(y|x)$$

- Decoding the best max-marginal at each variable node gives the MAP
- With cycles: not guaranteed to converge, and even if it does, no relation with LP-MAP

What Kind of Local Decoding Do We Need?



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AD^3 (Martins et al., 2011a)	QP/MAP

TRW-S (Kolmogorov, 2006)

Same rationale as sum-product TRBP: cover the graph with spanning trees, and compute messages using edge appearance probabilities

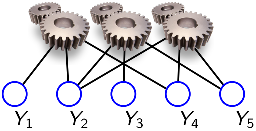
Only differences:

- Replace $\sum$ with max
- Messages need to be computed *sequentially* for convergence

As max-product loopy BP, all is required is to compute *local max-marginals*

Under mild assumptions, gives the solution of LP-MAP

What Kind of Local Decoding Do We Need?



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Max-Product LP (Globerson and Jaakkola, 2008)

Derived by writing the dual of LP-MAP, and solving it with a **block coordinate descent algorithm**

The message updates need to be computed in a sequential schedule

Progress in the dual objective is monotonic

Drawback: since the dual is non-smooth, we may get stuck at a suboptimal point

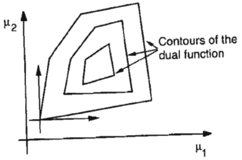


Figure 6.3.6. The basic difficulty with coordinate ascent for a nondifferentiable dual function. At some points it may be impossible to improve the dual function along any coordinate direction.

(From Bertsekas et al. (1999))

MPLP Messages

- **Variable-to-factor messages:**

$$m_{i \to s}(y_i) = \psi_i(y_i) \prod_{s' \in N(i) \setminus \{s\}} n_{s' \to i}(y_i)$$

- **Factor-to-variable messages:**

$$n_{s \to i}(y_i) = \frac{\max_{\boldsymbol{y}_s \sim y_i} \left(\psi_s(\boldsymbol{y}_s)^{1/|N(s)|} \prod_{j \in N(s)} m_{j \to s}(y_j)^{1/|N(s)|} \right)}{m_{i \to s}(y_i)}$$

Summary of MPLP

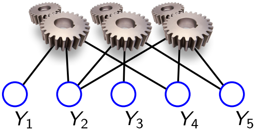
Advantages:

- Very simple to implement
- Handles structured and logic factors (only need to compute local max-marginals)
- Monotonically improves the dual
- No parameters to tune

Disadvantages:

- Can get stuck at a suboptimal solution (general problem with nonsmooth coordinate ascent)
- Messages are not computed in parallel (otherwise, may not converge)

What Kind of Local Decoding Do We Need?



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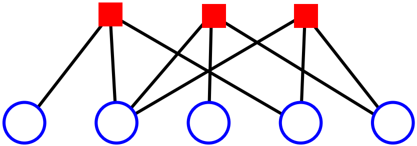
Summing Up Message-Passing

- BP algorithms and their variants can be used both for MAP and marginal decoding
- They need to compute local *marginals* (sum-product) or *max-marginals* (max-product)
- Always exact if the graph has no cycles; approximate otherwise
- They correspond to minimizing an energy approximation over the local polytope
- Some variants do convex approximations or compute upper bounds
- Two views of MAP decoding: (1) the near-zero temperature limit of marginal decoding; (2) a non-smooth optimization problem

Outline

Dual Decomposition

- Old idea in optimization (Dantzig and Wolfe, 1960; Everett III, 1963)
- First proposed by Komodakis et al. (2007) in computer vision
- Introduced in NLP by Rush et al. (2010) for model combination
- Successful in syntax, semantics, MT: Koo et al. (2010); Chang and Collins (2011); Martins et al. (2011b); Almeida et al. (2014); Martins and Almeida (2014), and many others.



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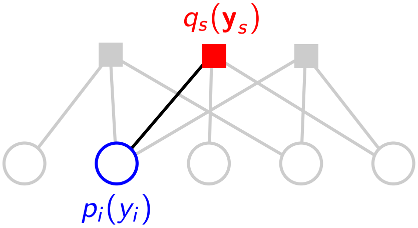

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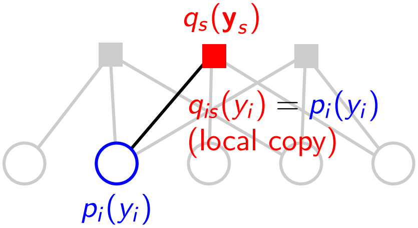

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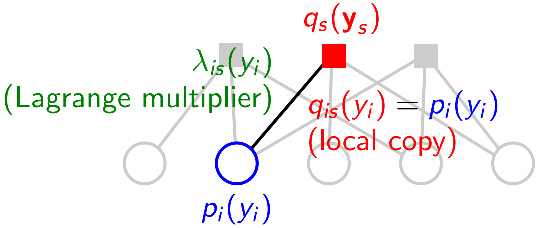

Recap: LP-MAP

Recall the LP-MAP problem:

$$\begin{aligned} \text{maximize} \quad & \sum_i \boldsymbol{\theta}_i^\top \boldsymbol{p}_i + \sum_s \boldsymbol{\theta}_s^\top \boldsymbol{q}_s \\ \text{subject to} \quad & \begin{cases} \boldsymbol{q}_s \in \Delta^{|\mathcal{Y}_s|}, \ \forall s \\ \boldsymbol{p}_i = \mathbf{M}_{is} \boldsymbol{q}_s, \ \forall i, s. \end{cases} \end{aligned}$$

(local polytope)

Matrix $\mathbf{M}_{is} \in \{0,1\}^{|\mathcal{Y}_i| \times |\mathcal{Y}_s|}$ represents the constraints $p_i(y_i) = \sum_{\boldsymbol{y}_s \sim y_i} q_s(\boldsymbol{y}_s)$

We'll reformulate this problem by:

1. Introducing copy variables $\boldsymbol{q}_{is} = \boldsymbol{p}_i$
2. Defining $\boldsymbol{\theta}_{is} := \boldsymbol{\theta}_i / |N(i)|$

Reformulation of LP-MAP

The problem becomes:

$$\text{maximize} \sum_s \left(\boldsymbol{\theta}_s{}^\top \boldsymbol{q}_s + \textstyle\sum_{i \in N(s)} \boldsymbol{\theta}_{is}{}^\top \boldsymbol{q}_{is} \right)$$

$$\text{subject to} \begin{cases} \boldsymbol{q}_s \in \Delta^{|\mathcal{Y}_s|}, \ \forall s \\ \boldsymbol{q}_{is} = \mathbf{M}_{is} \boldsymbol{q}_s, \ \forall i, s \\ \boldsymbol{q}_{is} = \boldsymbol{p}_i, \ \forall i, s. \end{cases} \qquad \textbf{(local polytope)}$$

By introducing Lagrange multipliers for the last constraints, we get the following **Lagrangian function**:

$$\mathcal{L}(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{\lambda}) = \sum_s \left(\boldsymbol{\theta}_s{}^\top \boldsymbol{q}_s + \textstyle\sum_{i \in N(s)} \boldsymbol{\theta}_{is}{}^\top \boldsymbol{q}_{is} \right) + \sum_{is} \boldsymbol{\lambda}_{is}{}^\top (\boldsymbol{p}_i - \boldsymbol{q}_{is})$$

Dual of LP-MAP

The dual problem is

$$\text{minimize} \sum_s g_s(\boldsymbol{\lambda}) \qquad \text{subject to } \boldsymbol{\lambda} \in \Lambda := \left\{ \boldsymbol{\lambda} \;\middle|\; \sum_{s \in N(i)} \boldsymbol{\lambda}_{is} = \mathbf{0} \right\}$$

where the $g_s(\boldsymbol{\lambda})$ are **local subproblems**,

$$\begin{aligned} g_s(\boldsymbol{\lambda}) \quad &:= \quad \max_{\bar{\boldsymbol{q}}_s \in \mathcal{Q}_s} \left(\boldsymbol{\theta}_s{}^\top \boldsymbol{q}_s + \textstyle\sum_{i \in N(s)} (\boldsymbol{\theta}_{is} + \boldsymbol{\lambda}_{is})^\top \boldsymbol{q}_{is} \right) \\ &= \quad \max_{\boldsymbol{y}_s \in \mathcal{Y}_s} \left(\boldsymbol{\theta}_s(\boldsymbol{y}_s) + \textstyle\sum_{i \in N(s)} (\boldsymbol{\theta}_{is}(y_i) + \boldsymbol{\lambda}_{is}(y_i)) \right) \end{aligned}$$

and $\bar{\boldsymbol{q}}_s \in \mathcal{Q}_s$ encodes the constraints $\left\{ \begin{array}{l} \boldsymbol{q}_s \in \Delta^{|\mathcal{Y}_s|} \\ \boldsymbol{q}_{is} = \mathbf{M}_{is} \boldsymbol{q}_s, \quad \forall i \in N(s). \end{array} \right.$

A subgradient can be computed by solving these local subproblems

Projected Subgradient (Komodakis et al., 2007)

initialize penalties $\boldsymbol{\lambda}$ to zero
repeat

until consensus (all $\boldsymbol{q}_{is} = \boldsymbol{p}_i$) or maximum number of iterations reached

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for each factor s **do**

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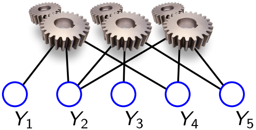
- Guaranteed to converge to an ϵ-accurate solution after at most $O(1/\epsilon^2)$ iterations
- **Problem:** too slow when there are many factors (Martins et al., 2011b)

What Kind of Local Decoding Do We Need?



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Accelerating Consensus

Two fundamental problems with the subgradient algorithm:

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Accelerated Gradient (Jojic et al., 2010)

Basic idea: make the dual objective smooth by adding an entropic perturbation with a near-zero ϵ temperature (also Johnson (2008))

The subproblems become local *marginal* computations instead of maximizations

With Nesterov's accelerated gradient method (Nesterov, 2005), the iteration bound goes from $O(1/\epsilon^2)$ to $O(1/\epsilon)$

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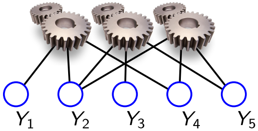
However: very sensitive to the temperature parameter

With low temperatures, may face numerical issues (in particular for some hard-constraint factors)

In practice, quite slow to take off (we'll see some plots later)

What Kind of Local Decoding Do We Need?



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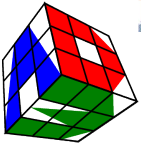
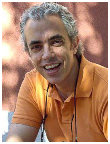
Alternating Directions Dual Decomposition (AD^3)


Based on the **alternating direction method of multipliers (ADMM):**

- an old method in optimization inspired by augmented Lagrangians (Gabay and Mercier, 1976; Glowinski and Marroco, 1975)
- a natural fit to consensus problems
- a natural "upgrade" of the subgradient algorithm (Boyd et al., 2011)

Augmented Lagrangian and ADMM

Basic idea: augment the Lagrangian function with a **quadratic penalty**

$$\begin{aligned}\mathcal{L}_\eta(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{\lambda}) &= \sum_s \left(\boldsymbol{\theta}_s{}^\top \boldsymbol{q}_s + \textstyle\sum_{i \in N(s)} \boldsymbol{\theta}_{is}{}^\top \boldsymbol{q}_{is} \right) + \sum_{is} \boldsymbol{\lambda}_{is}{}^\top (\boldsymbol{p}_i - \boldsymbol{q}_{is}) \\ &\quad - \frac{\eta}{2} \sum_{is} \|\boldsymbol{q}_{is} - \boldsymbol{p}_i\|^2\end{aligned}$$

Method of multipliers (super-linear convergence):

1. Maximize $\mathcal{L}_\eta(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{\lambda})$ jointly w.r.t. $\boldsymbol{p}$ and $\boldsymbol{q}$ (challenging)
2. Multiplier update: $\boldsymbol{\lambda}_{is} \leftarrow \boldsymbol{\lambda}_{is} - \eta(\boldsymbol{q}_{is} - \boldsymbol{p}_i)$

Alternating direction method of multipliers: replace step 1 by separate maximizations (first w.r.t. $\boldsymbol{q}$, then $\boldsymbol{p}$)

From Subgradient to AD^3 (Martins et al., 2011a)

initialize penalties $\boldsymbol{\lambda}$ to zero
repeat
  for each factor $s = 1, \ldots, S$ **do**
    $$\bar{\boldsymbol{q}}_s \leftarrow \arg\max_{\bar{\boldsymbol{q}}_s \in \mathcal{Q}_s} \boldsymbol{\theta}_s{}^\top \boldsymbol{q}_s + \sum_{i \in N(s)} (\boldsymbol{\theta}_{is} + \boldsymbol{\lambda}_{is})^\top \boldsymbol{q}_{is}$$
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- **faster consensus:** regularize $\boldsymbol{q}$-step towards average votes in $\boldsymbol{p}$

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- **faster consensus:** regularize $\boldsymbol{q}$-step towards average votes in $\boldsymbol{p}$
- **better stopping conditions:** keeps track of primal and dual residuals

Theoretical Guarantees of AD^3

Convergent in primal and dual (Glowinski and Le Tallec, 1989)

Iteration bound: $O(1/\epsilon)$ (cf. $O(1/\epsilon^2)$ for projected subgradient)

Inexact AD^3 subproblems: still convergent if residuals are summable (Eckstein and Bertsekas, 1992)

Always dual feasible: can compute upper bounds and embed in branch-and-bound toward *exact* decoding (Das et al., 2012)

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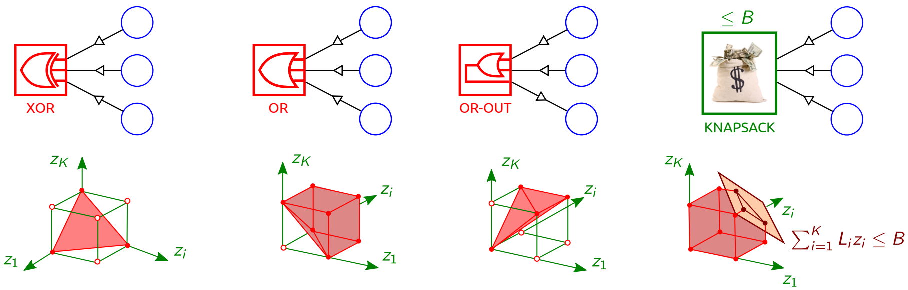
Still—very easy and efficient for logic and knapsack factors!

Projecting onto Hard Constraint Polytopes



- Martins et al. (2011a): logic factors can be solved in $O(K)$ time
- **Almeida and Martins (2013): same for *knapsack* factors!**

Structured Factors

What about structured factors?

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Projected subgradient handles these quite well

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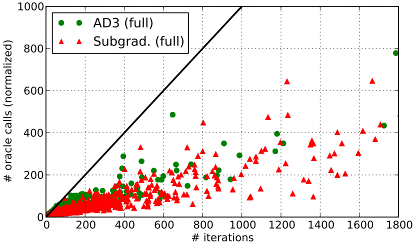
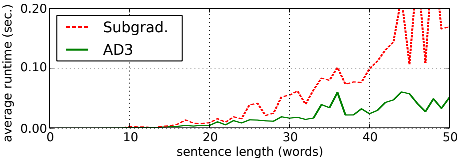
More info: Martins et al. (2014)

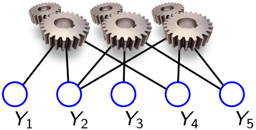
Runtime of AD^3 vs PSDD (Parsing)





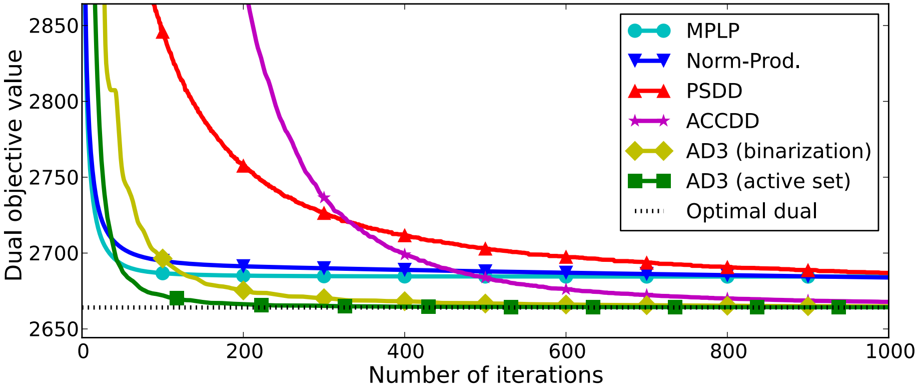
- **Caching** and **warm-starting** the subproblems reduces drastically the number of oracle calls—**huge speed-ups!!**
- **AD^3 faster to achieve consensus** (due to the quadratic penalty)

What Kind of Local Decoding Do We Need?



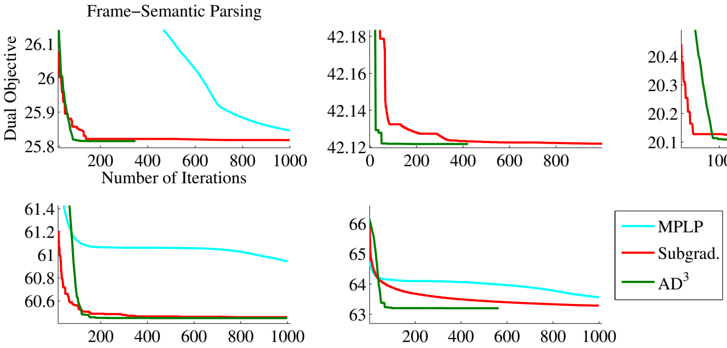
Algorithm	Local Operation
Sum-Prod. BP (Pearl, 1988)	marginals
TRBP (Wainwright et al., 2005)	marginals
Norm-Product BP (Hazan and Shashua, 2010)	marginals
Max-Prod. BP (Pearl, 1988)	max-marginals
TRW-S (Kolmogorov, 2006)	max-marginals
MPLP (Globerson and Jaakkola, 2008)	max-marginals
PSDD (Komodakis et al., 2007)	MAP
Accelerated DD (Jojic et al., 2010)	marginals
AD^3 (Martins et al., 2011a)	QP/MAP

Example: Potts Grid (20×20, 8 states)



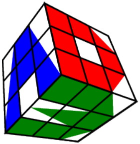
- A. Martins, M. Figueiredo, P. Aguiar, N. Smith, E. Xing (2014).
  AD^3: Alternating Directions Dual Decomposition for MAP Inference in Graphical Models.
  Journal of Machine Learning Research (to appear).

Example: Frame-Semantic Parsing



- Embedded in a branch-and-bound procedure for *exact* decoding
- D. Das, A. Martins, N. Smith.
  "An Exact DD Algorithm for Shallow Semantic Parsing with Constraints."
  *SEM Workshop, 2012.

Try It Yourself: AD^3 Toolkit



- Freely available at: *http://www.ark.cs.cmu.edu/AD3*
- Implemented in C++, includes a Python wrapper (thanks to Andy Mueller)
- Implements MPLP, PSDD, AD^3 for arbitrary factor graphs
- Many built-in factors: logic, knapsack, dense, and some structured factors
- You can implement your own factor (only need to write a local MAP decoder!)
- Toy examples included (parsing, coreference, Potts models)

Summing Up Dual Decomposition

- Dual decomposition is a general optimization technique that splits the dual into several subproblems (one per factor) that must agree on overlaps
- This can be used to solve LP-MAP
- We discussed three variants: subgradient (PSDD), accelerated gradient (ADD), and alternating directions (AD^3)
- The algorithms are convergent and retrieve the true MAP if the graph has no cycles; they also give certificates when the solution of LP-MAP equals the MAP
- For PSDD and AD^3 only local maximizations are necessary; ADD requires computing marginals

Outline

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We'll discuss three applications:

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- Compressive Summarization
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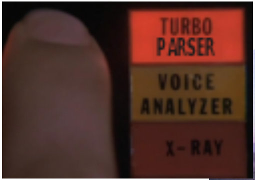

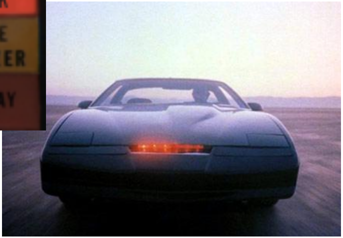

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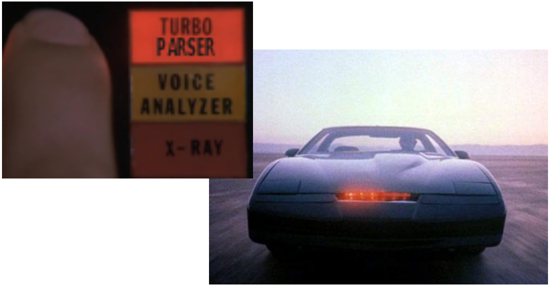

- **A parser that runs inference in factor graphs, ignoring global effects caused by loops (Martins et al., 2010)**
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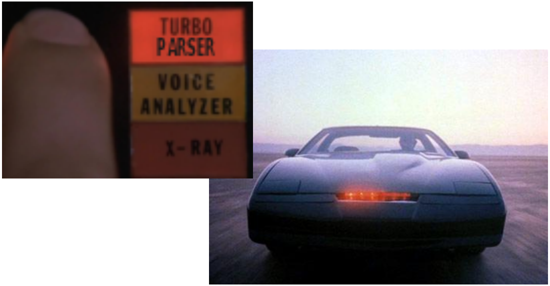

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- **Next:** we speed up turbo parsers via AD^3 w/ active set

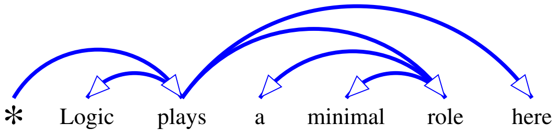
Recent Paper



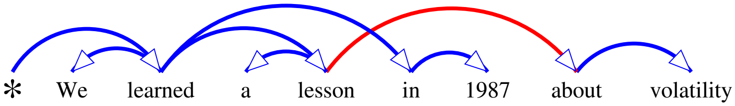
- André F. T. Martins, Miguel B. Almeida, Noah A. Smith. "Turning on the Turbo: Fast Third-Order Non-Projective Turbo Parsers." ACL 2013 Short Paper.

An Important Distinction

- A projective tree:



- A non-projective tree:



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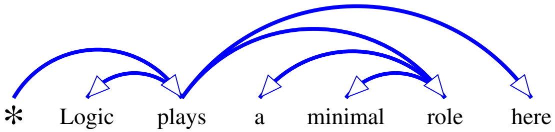

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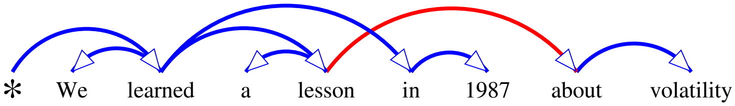


This talk: we allow non-projective trees.

Suitable for languages with flexible word order (Dutch, German, Czech,...)

First-Order Scores for Arcs

* We learned a lesson in 1987 about volatility

Second-Order Scores for Consecutive Siblings

* We learned a lesson in 1987 about volatility

Second-Order Scores for Grandparents

* We learned a lesson in 1987 about volatility

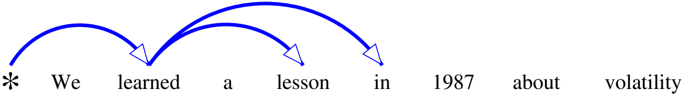
Scores for Arbitrary Siblings

* We learned a lesson in 1987 about volatility

Scores for Head Bigrams

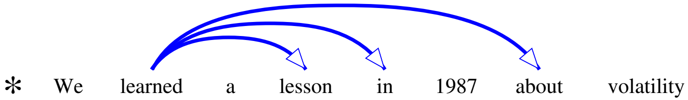
* We <u>learned</u> <u>a</u> lesson in 1987 about volatility

Third-Order Scores for Grand-siblings



Used by Koo and Collins (2010) for *projective* parsing.

Third-Order Scores for Tri-siblings



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Decoding

How to deal with all these parts?

- Dynamic programming only available for the *projective* case...

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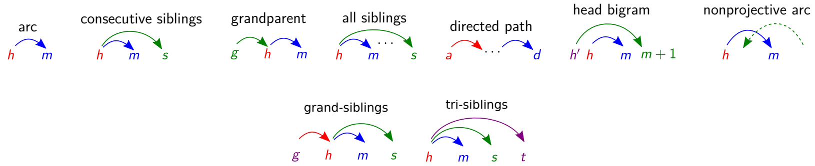
- Beyond arc-factored models, non-projective parsing is **NP-hard** (McDonald and Satta, 2007)
- **Need to embrace approximations!**

Approximate Dependency Parsers



	parser	AF	CS	G	AS	DP	HB	NPA	GS	TS
McDonald et al. (2006)	projective + greedy	✓	✓							
Smith et al. (2008)	loopy BP	✓	✓	✓	✓					
Martins et al. (2010)	LP solver	✓		✓	✓			✓		
Koo et al. (2010)	dual decomp.	✓	✓							
Martins et al. (2011)	AD^3	✓	✓	✓	✓	✓	✓	✓		
Martins et al. (2013)	AD^3 & active set	✓	✓	✓	✓		✓		✓	✓

Factor Graph Representation

- Variables nodes for **dependency arcs**, linked to a tree constraint
- Head automata for **consecutive siblings and grandparents** (as in Smith and Eisner (2008); Koo et al. (2010))
- Pairwise factors for **arbitrary siblings** (as Martins et al. (2011b))
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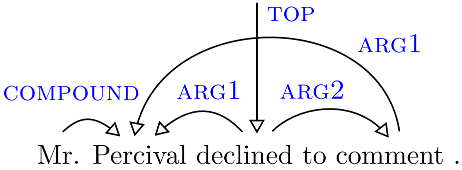
We solve the LP-MAP relaxation with AD^3.

Parsing Accuracies/Runtimes

SOTA accuracies for the largest non-projective datasets (CoNLL-2006 and CoNLL-2008):

Extension: Broad-Coverage Semantic Parsing

Same idea applied to **semantic role labeling**.



Best results in the SemEval 2014 shared task:

- André F. T. Martins and Mariana S. C. Almeida.
  "Priberam: A Turbo Semantic Parser with Second Order Features."
  SemEval 2014.

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Multi-Document Summarization

Map a set of related **documents** to a brief **summary.**

CNN

Obama hopes for 'continued progress' in Myanmar

STORY HIGHLIGHTS

- Obama meets with pro-democracy icon Aung San Suu Kyi and Myanmar's president
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"Here, through so many difficult years, is where she has displayed such unbreakable courage and determination," Obama told reporters, standing next to his fellow Nobel peace laureate. "It is here where she showed that human freedom and human dignity cannot be denied."

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The country, which is also known as Burma, was ruled by military leaders until early 2011 and for decades was politically and economically cut off from the rest of the world.

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(CNN) -- Barack Obama met with Nobel Peace Prize winner Aung San Suu Kyi at her home in Myanmar on Monday, praising her "courage and determination" during a historic visit to the once repressive and secretive country.

The first sitting U.S. president to visit Myanmar, Obama urged its leaders, who have embarked on a series of far-reaching political and economic reforms since 2011, not to extinguish the "flickers of progress that we have seen."

Obama said that his visit to the lakeside villa where the pro-democracy icon spent years under house arrest marked a new chapter between the two countries.

"Here, through so many difficult years, is where she has displayed such unbreakable courage and determination," Obama told reporters, standing next to his fellow Nobel peace laureate. "It is here where she showed that human freedom and human dignity cannot be denied."

Myanmar Obama visit

The country, which is also known as Burma, was ruled by military leaders until early 2011 and for decades was politically and economically cut off from the rest of the world.

Suu Kyi acknowledged that Myanmar's opening up would be difficult.

The New York Times

YANGON, Myanmar — President Obama journeyed to this storied tropical outpost of pagodas and jungles on Monday to "extend the hand of friendship" as a land long tormented by repression and poverty begins to throw off military rule and emerge from decades of isolation.

The visit was intended to show support for the reforms put in place by Thein Sein's government since the end of military rule in November 2010.

Activists have warned that the visit may be too hasty - political prisoners remain behind bars and ethnic conflicts in border areas are unresolved.

BBC NEWS

For given **summary size**, easier to be **informative**, but harder to be **grammatical**.

Compressive Summarization as Global Optimization

- Indicator variables for every word of the nth sentence, $\boldsymbol{z}_n := \langle z_{n,\ell} \rangle_{\ell=1}^{L_n}$

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- Indicator variables for every word of the nth sentence, $\boldsymbol{z}_n := \langle z_{n,\ell} \rangle_{\ell=1}^{L_n}$
- Summary length must not exceed the **budget** (B words)
- Quality function rewards *global* **informativeness** (through $g(\boldsymbol{z})$)...
- ... but also *local* **grammaticality** (through $h_n(\boldsymbol{z}_n)$):

$$\begin{array}{ll} \text{maximize} & g(\boldsymbol{z}) + \sum_{n=1}^{N} h_n(\boldsymbol{z}_n) \\ \text{s.t.} & \sum_{n=1}^{N} \sum_{\ell=1}^{L_n} z_{n,\ell} \leq B. \end{array}$$

Grammaticality: Sentence Compression Model

Inspired by Knight and Marcu (2000)'s word deletion model

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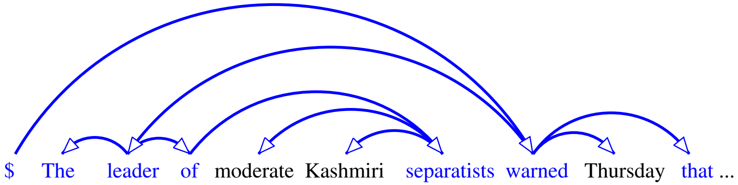
Our model factors over dependency arcs:



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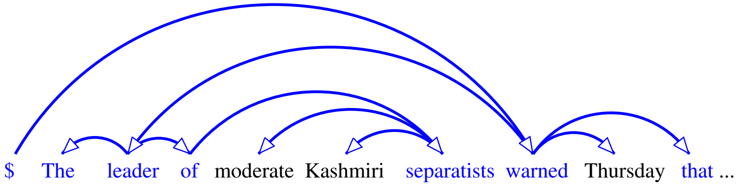

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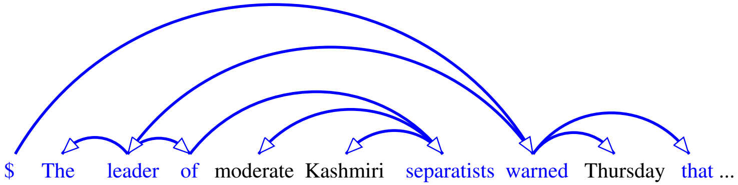

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A structured factor, locally decodable with dynamic programming.

Informativeness: Coverage Model

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- Define **relevance scores** for each concept (linear feature-based model)
- Define $g(\boldsymbol{z})$ as sum of scores for each **concept** in the summary

Graphical Model for Our Compressive Summarizer

Budget

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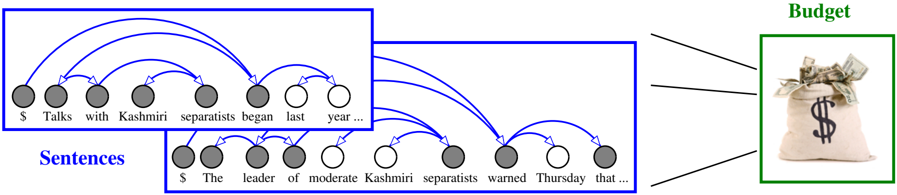

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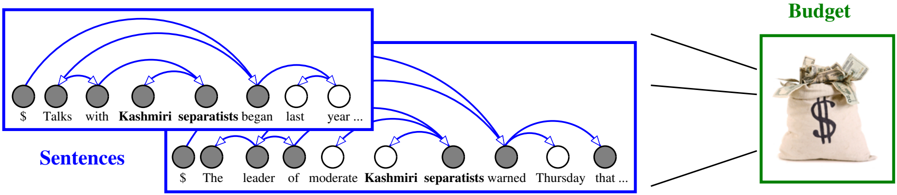

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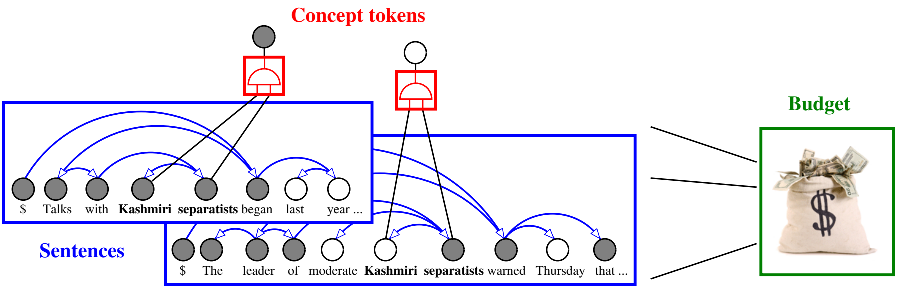

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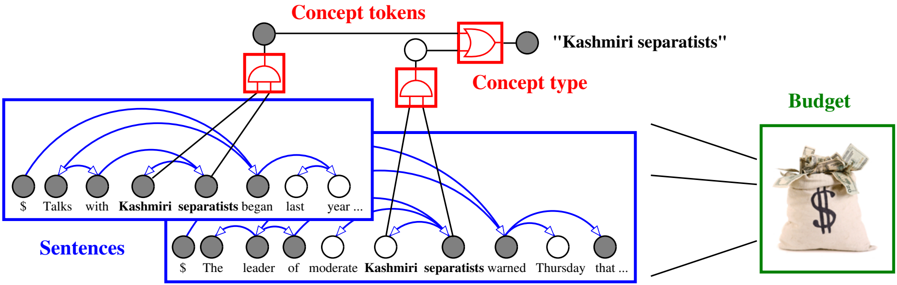

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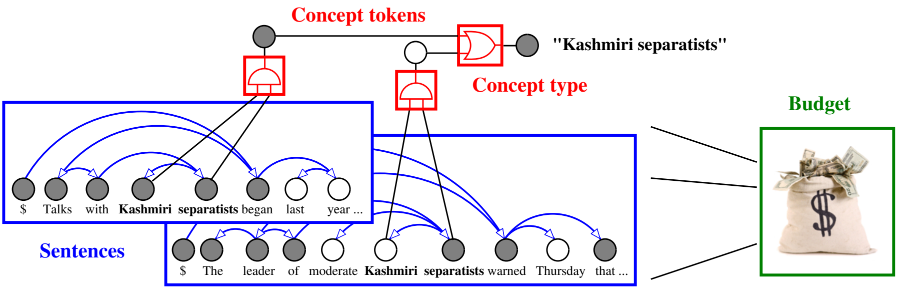

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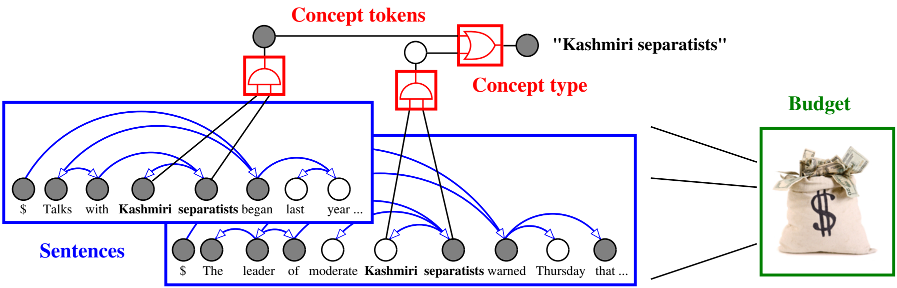

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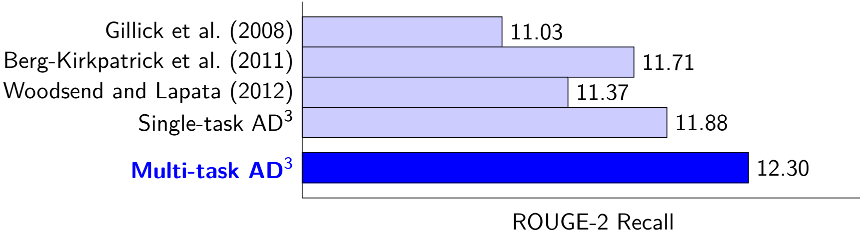
Multi-task learning: user-generated data (Simple English Wikipedia) along with manual abstracts and compressive summaries

Results on TAC-2008 Dataset

- Better informativeness (without sacrificing grammaticality):



- Averaged runtimes per summarization problem (10 documents):

Solver	Runtime (sec.)	ROUGE-2
ILP Exact, GLPK	10.394	12.40
LP-Relax., GLPK	2.265	12.38
AD^3 (1,000 its.)	**0.406**	**12.30**
Extractive (ILP)	0.265	11.16

Applications

We'll discuss three applications:

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Recent Paper

  

- Mariana S. C. Almeida, Miguel B. Almeida and André F. T. Martins. "A Joint Model for Quotation Attribution and Coreference Resolution." EACL 2014.

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Two consecutive quotes are often from co-referent speakers:

> *English novelist Dorothy L. Sayers described ringing as a "passion that finds its satisfaction in mathematical completeness and mechanical perfection."*
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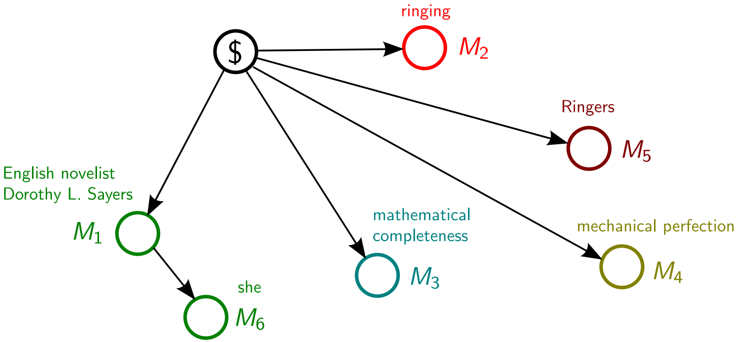
Coreference Tree (Denis and Baldridge, 2008; Fernandes et al., 2012; Durrett and Klein, 2013)



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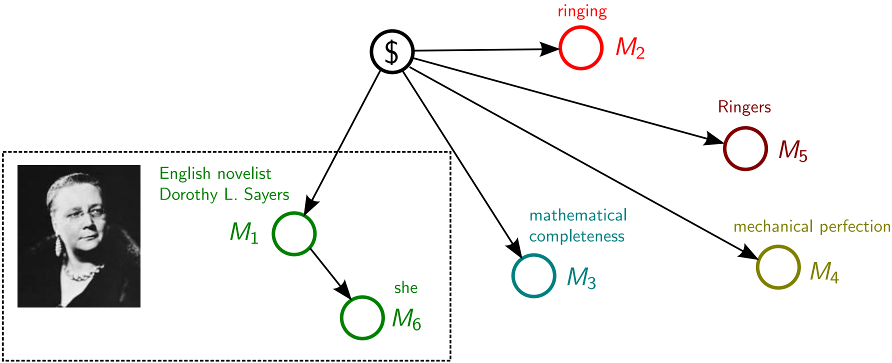

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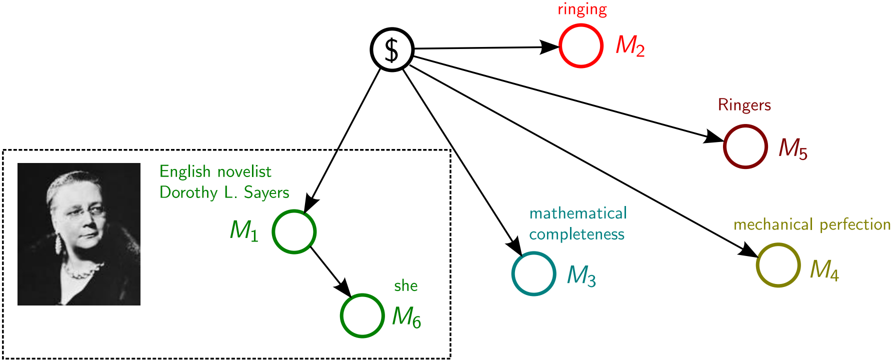
From Coreference to *Quotation-Coreference* Trees (Almeida et al., 2014)

- Include mention nodes **and quotation nodes**
- Quotation nodes have to be leaves
- Subtrees coming out from the root induce entity clusters along with their quotes: **entity-based quotation attribution**

From Coreference to *Quotation-Coreference* Trees (Almeida et al., 2014)



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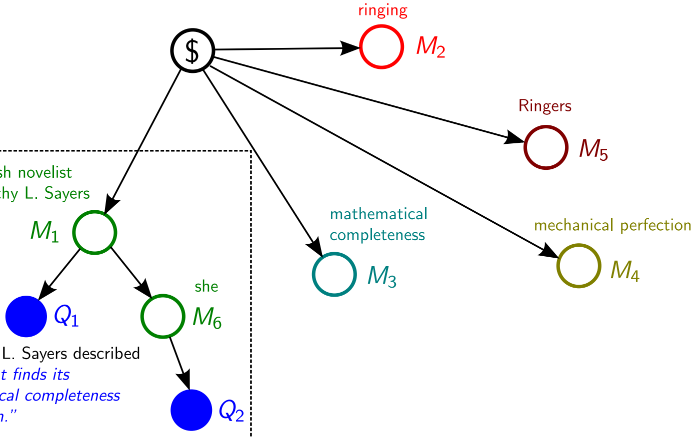

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Beyond Arc Scores

The simplest coreference models (e.g., the SURFACE model of Durrett and Klein (2013)) are **arc-factored**

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These scores require knowing if pairs of nodes are in the same subtree.

Logic Program

- **Arc variables:** each node (except the root) has exactly one parent

$$\sum_{i=0}^{j-1} a_{i \to j} = 1, \quad \forall j \neq 0$$

- **Path variables**: paths propagate through arcs

$$\pi_{i \to^* i} = 1, \quad \forall i, \quad \pi_{i \to^* k} = \bigvee_{i < j \leq k} (a_{i \to j} \wedge \pi_{j \to^* k}), \quad \forall i, k$$

- **Pair variables:** nodes k and ℓ are in the same subtree if they have a common ancestor which is not the root

$$p_{k,\ell} = \bigvee_{i \neq 0} (\pi_{i \to^* k} \wedge \pi_{i \to^* \ell}), \quad \forall k, l.$$

Experiments

Datasets:

- WSJ portion of the Ontonotes (597 documents); same splits as CoNLL 2011 shared task
- Quotation annotations of the PARC dataset (Pareti, 2012; O'Keefe et al., 2012)

Coreference evaluation metrics: average between MUC, B^3, $CEAF_e$

Quotation evaluation metrics:

- **Representative speaker match (RSM):** # matches to representative (non-pronominal) mention of the gold speaker's entity
- **Entity cluster F_1 (ECF$_1$):** F_1 score between the predicted and gold speaker entity (sets of mentions)

Results

Coreference Resolution:

	MUC F_1	BCUB F_1	CEAFE F_1	Avg.
Durrett and Klein (2013) (SURFACE)	**58.87**	62.74	45.46	55.7
QUOTE/COREF INDEPENDENT	57.89	62.50	45.48	55.3
JOINT SYSTEM	58.78	**63.79**	**45.50**	**56.0**

Quotation attribution:

	RSM	ECF_1
QUOTEONLY	49.4%	41.2%
QUOTEAFTERCOREF	64.6%	70.0%
QUOTE/COREF INDEPENDENT	74.7%	73.7%
JOINT SYSTEM	**76.6%**	**74.1%**

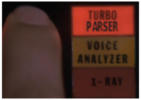
Outline

Conclusions

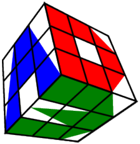
- Many structured problems in NLP are NP-hard or expensive (constrained models, diversity, combination of structured models)
- Often they can be approximately decoded via Linear Programming (e.g., by relaxing an ILP)
- The structure inherent to these problems can be represented with a factor graph
- Message-passing and dual decomposition algorithms can solve these LPs efficiently, exploiting the structure of the graph
- Conceptually: approximate *global* decoding by invoking only *local* decoders (local maximizations, marginals, max-marginals, QPs, ...)
- AD^3 is faster than the subgradient algorithm both in theory and in practice, and requires the same local decoders
- SOTA results in several applications (turbo parsing, summarization, joint coref and quotation attribution)

Thank you!

The syntactic/semantic parser and AD^3 are freely available at:



http://www.ark.cs.cmu.edu/TurboParser
http://www.ark.cs.cmu.edu/AD3



ARK lti Carnegie Mellon

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